

Basic principles of particle accelerator Physics

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- Introduction: Energy and Forces
- Electric fields:
 - I Acceleration by DC voltages
 - II Acceleration by time-varying fields
- Magnetic fields: transverse motion
- What's next?
- Conclusions

Introduction: Energy and Forces

- The study of the basic building blocks of matter and the forces which act between them is a fundamental area of Physics. The structures under investigations are extraordinary small ($\sim 10^{-15}$ m)
- In order to perform experiments at this scale probes with high spatial resolution are needed (visible light $\lambda \approx 500$ nm \Rightarrow wholly inadequate) \Rightarrow High energy particle beams are essential to this field of experimental physics
- But many other applications of **accelerator science** have been developed during the last XX century, hadron therapy, biophysics, material science, radiopharmaceuticals, nuclear energy production and waste incineration, synchrotron light sources, spallation sources, radiography, ion implantation and surface metallurgy, inertial fusion drivers, PET & NMR scanners...
- In which way is the probe wavelength related to particle's energy?
Through the De Broglie wavelength

$$\lambda_B = \frac{h}{p} = \frac{hc}{E}$$

Introduction: Energy and Forces

- Another important aspect of elementary particle physics is the production of new particles. The amount of energy needed to produce a particle follows directly from the Einstein's relation

$$E_0 = mc^2$$

Some few examples of rest energies of particles investigated nowadays:

Particle	Symbol	E_0 [MeV]
proton	p	938
b quark	b	4735
vector boson	Z_0	91190
t quark	t	174000

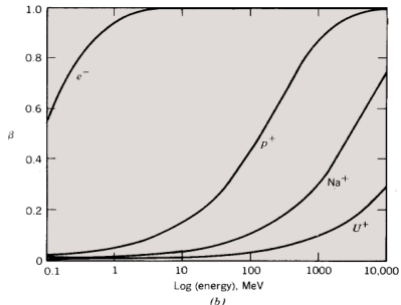
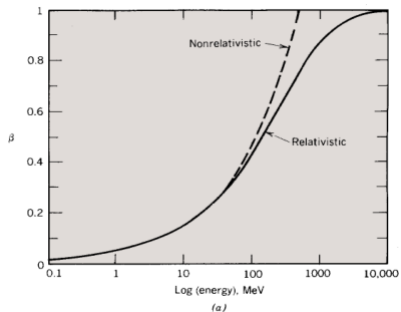
- Since the velocity v of elementary particles studied in collisions is generally close to $c = 2.997925 \cdot 10^8$ m/s, the energy must be written in the relativistically invariant form

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

$p = mv = \gamma m_0 v$ is the only free parameter! ($\gamma = \frac{1}{\sqrt{1-\beta^2}}$, $\beta = \frac{v}{c}$)

- The increase in energy E is the same as the increase in particle momentum p . This latter can only be changed by the action of a force \vec{F} on the particle, according to Newton's second law $\dot{\vec{p}} = \vec{F}$

Introduction: Energy and Forces



- Modern accelerators can accelerate particles to speeds very close to that of light
- At low energies (Newton), the velocity of the particle increases with the square root of the kinetic energy
- At relativistic energies (Einstein), the velocity increases very slowly asymptotically approaching that of light
- It is as if the velocity of the particle ‘saturates’
- However, one can pour more and more energy into the particle, giving it a shorter λ_B , so that it probes deeper into the sub-atomic world

Introduction: Energy and Forces

- Back to force, in order to reach high kinetic energies, a sufficiently strong force must be exerted on the particle for a sufficient period of time.
- You should be able to recognize, from the previous talk by Prof. Testa, how many forces does Nature offer us and of which type (what kind of particles do they affect, their strength and range)
- The only possible choice left is the **electromagnetic force**

When a particle of velocity \vec{v} passes through a volume containing a magnetic field \vec{B} and an electric field \vec{E} it is acted upon by the **Lorentz force**

$$\vec{F} = e \cdot (\vec{E} + \vec{v} \times \vec{B})$$

As the particle moves from point \vec{r}_1 to \vec{r}_2 its energy changes by the amount

$$\Delta E = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = e \int_{\vec{r}_1}^{\vec{r}_2} (\vec{E} + \vec{v} \times \vec{B}) \cdot d\vec{r}$$

During the motion the path element $d\vec{r}$ is always parallel to the velocity vector $\vec{v} \Rightarrow \vec{v} \times \vec{B} \perp d\vec{r} \Rightarrow (\vec{v} \times \vec{B}) \cdot d\vec{r} \equiv 0$. The magnetic field \vec{B} does not change the energy of the particle! Acceleration involving an increase in energy can only be achieved by the use of electric fields:

$$\Delta E = e \int_{\vec{r}_1}^{\vec{r}_2} \vec{E} \cdot d\vec{r} = eU$$

- Magnetic fields do not contribute to the energy of the particle, but they play a very important role when forces are required which act perpendicular to the particle's direction of motion \Rightarrow particle beams steering, bending and focusing! (we will have a look at magnetic fields more specifically in a while...)
- Accelerator Physics is thus concerned with the two problems: the acceleration and steering of particle beams
- Both processes rely on the electromagnetic force \Rightarrow **Maxwell equations** and **special theory of relativity** results are of **FUNDAMENTAL** importance

Electric fields: acceleration techniques

Use of the electric field

$$\vec{E} = -\nabla\phi - \frac{\partial\vec{A}}{\partial t}$$

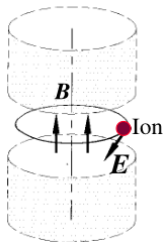


Acceleration by DC voltages:

- Cockcroft & Walton rectifier generator
- Van de Graaf electrostatic generator
- Tandem electrostatic accelerator

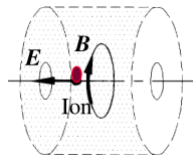
Acceleration by time-varying fields:

Betatron or
'unbunched'
acceleration

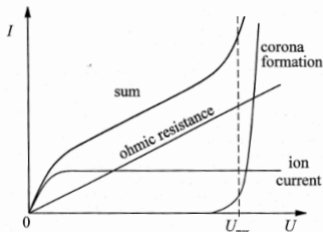
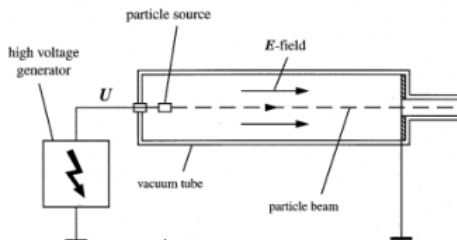


Resonant or
'bunched'
acceleration

- Linac
- Synchrotron
- Cyclotron
- RFQ
- Colliders

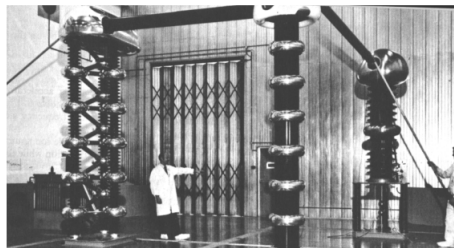
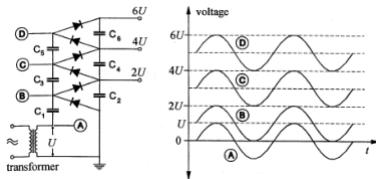


Acceleration by DC voltages: the direct-voltage accelerator



- Constant \vec{E} between two electrodes, produced by HV generator
- One of the electrodes contains the particle source (thermoionic cathode in the case of e^- beams)
- p or heavy ions can be extracted from the gas phase by DC or HFV to ionize a very rarified gas and so produce a plasma inside the source
- Good vacuum in the accelerating region, in order to avoid particle collisions with residual gas molecules
- Still nowadays employed in research and technology (screens, oscilloscopes)
- Particle energies very limited (the maximum energy achievable is directly proportional to the maximum voltage developed, voltages of few MV are technically possible \Rightarrow maximum energy of few MeV)

Acceleration by DC voltages: Cockroft-Walton cascade generator

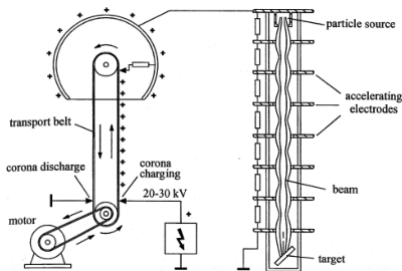


- HV generator based on a system of multiple rectifiers
- At point A a transformer produces $U(t) = U \sin \omega t$
- The first rectifying diode ensures at point B U never goes negative $\Rightarrow C_1$ charges up to U
- At point B the voltage oscillates between 0 and $2U$. C_2 is charged up to $2U$. The third diode ensures in potential in C does not fall below $2U$ and so on
- Maximum achievable voltage is $2nU$, with n number of the rectifier stages
- Actually I must always be drawn from the generator. Capacitors are always slightly discharged leading to a lower generated voltage than expected:

$$U_{tot} = 2nU - \frac{2\pi I}{\omega C} \left(\frac{2}{3}n^3 + \frac{1}{4}n^2 + \frac{1}{12}n \right)$$

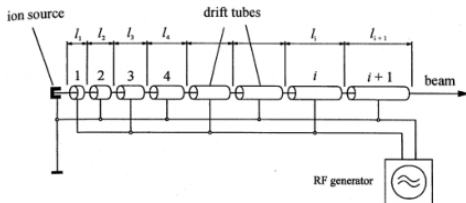
- Voltages up to about 4 MV can be reached

Acceleration by DC voltages: Van de Graaf accelerator



- Key element: belt made of isolating material, looped around two rollers
- Charge produced in corona formation around a sharp electrode is transferred onto the belt, which itself charges the isolated conducting dome until the critical voltage is reached.
- The dome is connected to the particle accelerator, containing the source and consisted of a large number of circular electrodes arranged in a line, connected by resistors \Rightarrow spark discharge risk highly reduced
- These generators can produce voltages of up to 2 MV or, if placed in tanks with an insulating gas (SF_6) at VHP (1 MPa), even up to 10 MV

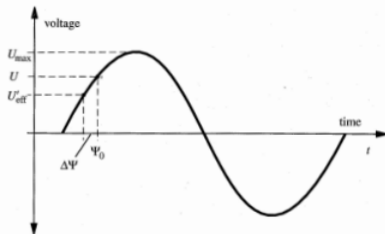
Acceleration by time-varying fields: LINAC



- Series of metal drift tubes arranged along the beam axis and connected, with alternating polarity, to a RF supply: $U(t) = U_{max} \sin \omega t$
- After the i^{th} drift tube the particles of charge q have reached an energy $E_i = iqU_{max} \sin \Psi_0$, with Ψ_0 being the average phase of the RF voltage particles see crossing the gaps
- E proportional to the number of stages i traversed by the particles
- $U < U_{max}!$ \Rightarrow **high particle energies without voltage discharge**
- Assuming $v \ll c$, in the i^{th} drift tube the velocity V_i is reached, thus $E_i = \frac{1}{2}mv_i^2$
- Through one drift section RF voltage moves through exactly $\tau_{RF}/2$. This fixes the separation between i^{th} and $(i+1)^{th}$ gaps to be

$$l_i = \frac{v_i \tau_{RF}}{2} = \frac{v_i}{2\nu_{RF}} = \frac{1}{\nu_{RF}} \sqrt{\frac{iqU_{max} \sin \Psi_0}{2m}}$$

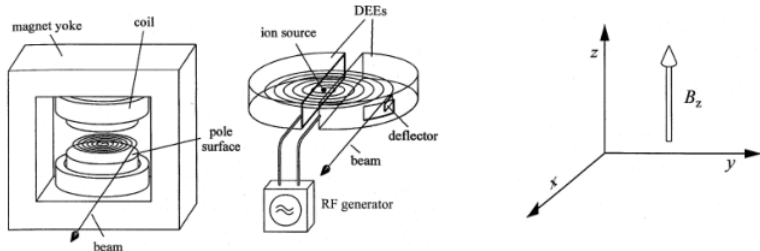
Acceleration by time-varying fields: LINAC



- The energy transferred to the particles depends critically on the U_{max} and Ψ_0
- When a very large number of stages are used, a small deviation from U_{max} means that the particle velocity no longer matches the design velocity fixed by $l_i \Rightarrow$ phase shift relative to the RF voltage \Rightarrow synchronization lost between particle motion and RF
- Any cure? **Phase focusing:** $\Psi_0 < \pi/2$ and $U_{eff} < U_{max}$

Assume a particle gains too much energy in a preceding stage, so travelling faster than an ideal particle and arriving earlier. It feels an average RF $\Psi = \Psi_0 - \Delta\Psi$, thus $U'_{eff} = U_{max} \sin(\Psi_0 - \Delta\Psi) < U_{max} \sin \Psi_0$. The particle thus gain less energy and slows down again until it returns to the nominal velocity. The opposite happens for too slow particles.

Acceleration by time-varying fields: Cyclotron



- Driving the particles around a circular path, using the same accelerating structure many times is a desirable alternative to costly linacs (cost growing with energy because of the number of accelerating cavities)
- Cyclotrons use an iron magnet $B \approx 2$ T between its two round poles
- Particles circulate in a plane between the poles

For $\vec{B} \equiv (0, 0, B_z)$ the equations of motions comes from the Lorentz force, setting $\vec{E} = 0$:

$$\vec{F} = \dot{\vec{p}} = \frac{d}{dt}(m\vec{v}) = e\vec{v} \times \vec{B}$$

Acceleration by time-varying fields: Cyclotron

Being the motion confined in $x - y$ plane:

$$\vec{p} \equiv (p_x, p_y, 0) = m(v_x, v_y, 0) \Rightarrow \dot{\vec{p}} = e(v_y B_z, -v_x B_z, 0)$$

Differentiating again wrt time we obtain the equations of motion:

$$\ddot{v}_x + \frac{e^2}{m^2} B_z^2 v_x = 0$$

$$\ddot{v}_y + \frac{e^2}{m^2} B_z^2 v_y = 0$$

with solutions

$$v_x(t) = v_0 \cos \omega_c t$$

$$v_y(t) = v_0 \sin \omega_c t$$

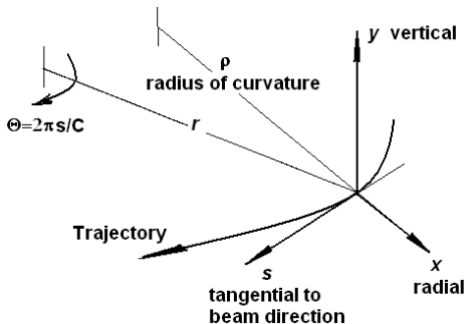
The particles follow a circular orbit between the poles with a revolution frequency (also known as the **cyclotron frequency**)

$$\omega_c = \frac{e}{m} B_z$$

- ω_c does not depend on the particle velocity. This is because as the energy increases, the orbit radius and hence the circumference along which the particles travel increase proportionally
- Higher velocity is compensated for by a larger radius, provided m remains constant \Rightarrow non-relativistic particles (for p, D, α @ 22 MeV, $v \approx 0.15 c$)

Magnetic fields: co-ordinate system

Local curvilinear co-ordinate system that follows the central orbit of the beam



- The tangential co-ordinate, which is directed along the central orbit, is designated as 's' (distance along the beam)
- Note that 'z' will be used as a general co-ordinate that can be either 'x' (horizontal) or 'y' (vertical)

Magnetic fields: terminology

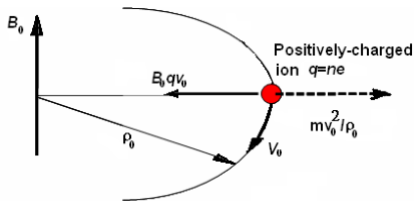
- In general, an accelerator lattice comprises a series of magnetic and/or electrostatic and/or electromagnetic elements separated by field-free, drift spaces
- In most cases, the lattice is dominated by magnetic dipoles and quadrupoles that constitute what is called the *linear lattice*. Quadrupole and higher-order lenses are usually centered on the orbit and do not affect the geometry of the accelerator
- The trajectory followed by the reference particle is known as the *central orbit* or *equilibrium orbit*
- In a ‘ring’ lattice, the enforced periodicity defines the equilibrium orbit unambiguously and obliges it to be closed. For this reason, it is often called the *closed orbit*
- Particles of the same momentum as the reference one, but with small spatial deviations, will oscillate about the equilibrium orbit: *betatron oscillations*
- Particles with a different momentum will have a different equilibrium orbit that will be referred to as an *off-momentum* or *off-axis equilibrium orbit*. Off-momentum particles with small spatial errors will perform betatron oscillations about their off-momentum equilibrium orbit

- An exact determination of the equilibrium orbit and the focusing along that orbit are difficult, if not impossible
- Measuring the beam position in an existing lattice or tracking through a field map are both techniques of limited precision
- To make calculations more tractable, while still providing a reasonably accurate picture, the '*hard-edge*' magnet model has been developed

Magnetic fields: ‘hard-edge’ model

- Dipoles, quadrupoles and solenoids are replaced by ‘blocks’ of field that are uniform in the axial direction within the block and zero outside
 - Multipole lenses are replaced by point kicks
 - The link to the real world is established equating the field integral in the model to the field integral in the real world magnet
- I The model must respect Maxwell’s equations to ensure that phase space is conserved and the model violates no fundamental principles
 - II The beam betatron oscillations should have a wavelength that is much longer than the fringe-field regions. This factor determines the level of convergence between what the model predicts and what actually occurs
 - III In some instances fringe-fields corrections may be applied to improve this agreement
 - IV Cyclotron motion in the uniform blocks of dipole field underpins the geometry of the ‘hard-edge’ model

Magnetic fields: more on cyclotron motion



- Let's equate expressions for the centripetal force: $B_0 q v_0 = -\frac{m v_0^2}{\rho_0}$
- This leads to a universally-used 'engineering' formula, which relates the momentum of the particle to its *Magnetic Rigidity* (reluctance to be deviated by the magnetic field)

$$|B_0 \rho_0| [\text{Tm}] = \left(\frac{3.3356}{n} \right) A |\vec{p}| [\text{GeV}/c]$$

- $q = ne$, A is the atomic mass number (for ions) and \vec{p} is the average momentum per nucleon so that $A p = m v_0$ (for ions)
- Since the formula is based on momentum, the non-linear effects of relativity are hidden

Magnetic fields: more on cyclotron motion

Cyclotron motion leads to a second universally-used ‘engineering’ formula, which relates the bending of the particle trajectory to its Magnetic rigidity:

$$\alpha[\text{rad}] = \frac{\int B_{\perp}[\text{T}]ds[\text{m}]}{|B_0\rho_0|[\text{Tm}]}$$

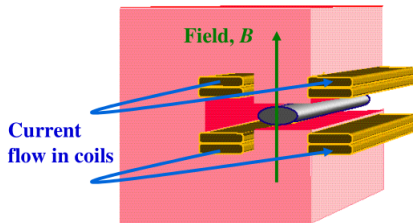
- The ‘hard-edge’ model is used for almost all lattice calculations
- In this model the central or equilibrium orbit is a stepwise progression of straight sections and circular arcs of cyclotron motion
- For a singly charged particle Magnetic Rigidity formula simplifies to

$$|B_0\rho_0|[\text{Tm}] = 3.3356|\vec{p}|[\text{GeV}/c]$$

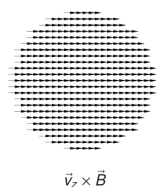
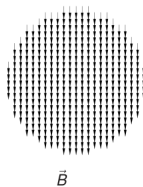
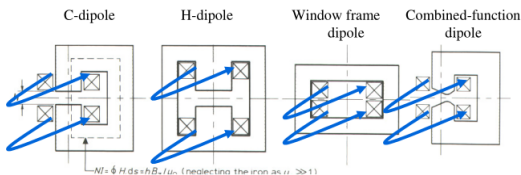
- To derive the angle formula:

$$\alpha = \frac{l}{\rho} \Rightarrow \alpha = \frac{l}{\rho} \frac{B\rho}{B\rho} \Rightarrow \alpha = \frac{\int Bds}{|B\rho|}$$

Magnetic fields: dipoles

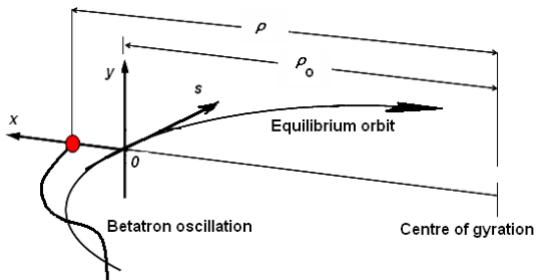


- ‘C-shaped’ dipole. Useful for injection, extraction and junctions in transfer lines



- Various dipole and combined function cross-sections

Magnetic fields: transverse motion



- The transverse motion in the lattice will be described by small perturbations from the central orbit that comprises a stepwise progression of straight sections and segments of cyclotron motion
- The transverse motion will be derived in a *local curvilinear co-ordinate system* (x, y, s) that follows the central orbit

Magnetic fields: transverse motion, bending plane

- Only magnetic elements are considered at this stage, momentum remains constant
- It is assumed that the deviation from the circular orbit will always be small and angular velocity can be approximated by v_0/ρ so that

$$F_\rho = m \frac{d^2 \rho}{dt^2} - m \frac{v_0^2}{\rho} = qv_0 B_y$$

Thus the magnetic deflection is considered as a ‘central force’ and is equated to the radial acceleration

- Two transformations will be used to introduce the local (x, y, s) co-ordinate system that follows the equilibrium orbit

$$\frac{d}{dt} \equiv v_0 \frac{d}{ds}$$

,

$$\rho = \rho_0 + x$$

to give

$$\frac{d^2 x}{ds^2} - \frac{1}{\rho_0 + x} = \frac{q}{mv_0} B_y$$

with the charge to mass ratio $\frac{q}{m} = \frac{ne}{Am} = -\frac{v_0}{B_0 \rho_0}$

Magnetic fields: transverse motion, bending plane

Now expand the field in a Taylor series up to the quadrupole component

$$B_y = \left\{ B_0 + \left(\frac{\partial B_y}{\partial x} \right)_0 x \dots \right\} = \{ B_0 - |B\rho| k x \dots \}$$

where

$$k = -\frac{1}{|B\rho|} \left(\frac{\partial B_y}{\partial x} \right)_0$$

is the *normalized gradient*.

Substituting for the field and remembering that $x \ll \rho$ gives

$$\frac{d^2x}{ds^2} + \left(\frac{1}{\rho_0^2} - k \right) x = 0$$

(Remember $\frac{1}{\rho_0+x} = \frac{1}{\rho_0} \left(1 - \frac{x}{\rho_0} + \dots \right)$)

- Note: A full derivation using Hamiltonian mechanics covering several pages comes to the same result!

Small increments in mass and velocity lead to (let's omit derivation, analogous to the previous one)

$$F_\rho = \frac{d}{dt} \left[(m + \Delta m) \frac{d}{dt} \rho \right] - (m + \Delta m) \frac{(v_0 + \Delta v)^2}{\rho} = q(v_0 + \Delta v) B_y$$

Now transform time t to distance s

$$\frac{d}{dt} \equiv (v_0 + \Delta v) \frac{d}{ds}$$

$$\frac{d^2 x}{ds^2} - \frac{1}{\rho_0 + x} = \frac{q}{(m + \Delta m)(v_0 + \Delta v)} B_y$$

To first order

$$\frac{1}{(m + \Delta m)(v_0 + \Delta v)} = \frac{1}{mv_0} \left(1 - \frac{\Delta m}{m} - \frac{\Delta v}{v_0} \right) = \frac{1}{p_0} \left(1 - \frac{\Delta p}{p_0} \right)$$

so that

$$\frac{d^2 x}{ds^2} + \left(\frac{1}{\rho_0^2} - k \right) x = \frac{1}{\rho_0} \frac{\Delta p}{p_0}$$

Basically the analysis is repeated, except that the magnetic fields has a different form,

$$F_y = \frac{d}{dt} \left[(m + \Delta m) \frac{dy}{dt} \right] = -q(v_0 + \Delta v) B_\rho$$

$$\frac{d^2 y}{ds^2} = - \frac{q}{(m + \Delta m)(v_0 + \Delta v)} B_\rho$$

Remember that to first order

$$\frac{1}{(m + \Delta m)(v_0 + \Delta v)} = \frac{1}{mv_0} \left(1 - \frac{\Delta m}{m} - \frac{\Delta v}{v_0} \right) = \frac{1}{p_0} \left(1 - \frac{\Delta p}{p_0} \right)$$

which gives

$$\frac{d^2 y}{ds^2} = \frac{1}{B_0 \rho_0} \left(1 - \frac{\Delta p}{p_0} \right) B_\rho$$

Magnetic fields: transverse motion, plane \perp to bending

Now expand the field and replace B_ρ by B_x ,

$$B_\rho \equiv B_x = \left(\frac{\partial B_x}{\partial y} \right)_0 y; \quad B_x = 0 \text{ for } y = 0$$

Substitution in the equation of motion gives

$$\frac{d^2 y}{ds^2} = -ky \left(1 - \frac{\Delta p}{p_0} \right)$$

We consider the ' $y\Delta p/p$ ' as second order and discard it to finish with

$$\frac{d^2 y}{ds^2} + ky = 0$$

- Note that $\Delta p/p$ has disappeared so this equation works (to first order) for on- and off- momentum particles. The k applies to gradient in combined-function dipoles. For a pure dipole, $k = 0$ and the dipole acts like a drift space!

Magnetic fields: transverse motion, summary

The equation of motion in a general form can be written as

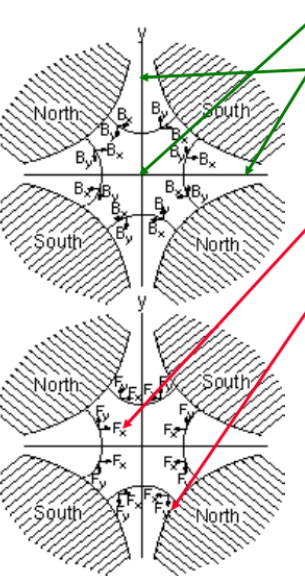
$$\frac{d^2z}{ds^2} + K_z(s)z = \frac{1}{\rho_0(s)} \frac{\Delta p}{p_0}$$

where z can be either x or y and $K_z(s)$ is the ‘focusing constant’ for the motion. In the plane perpendicular to the bending, $\rho = \infty$ and the RHS term is removed

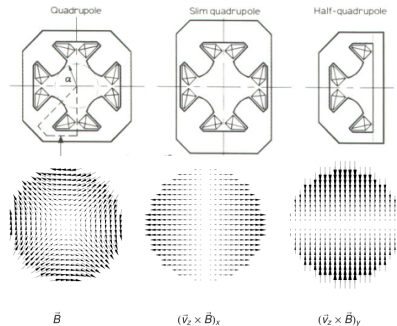
Element	K_x	K_y
Magnetic combined-function with horizontal bend	$\rho_0^{-2} - k$	k
Magnetic combined-function with vertical bend	$-k$	$\rho_0^{-2} + k$
Pure magnetic quadrupole	$-k$	k
Pure magnetic horizontal bend	ρ_0^{-2}	0
Pure magnetic vertical bend	0	ρ_0^{-2}
Drift space	0	0

- In fact, this covers almost 90% of all lattices. Note that K and ρ are functions of s . This is meant to indicate that these parameters change from one element to the next, but it is understood that they are constant within an element

Magnetic fields: quadrupoles



- ❖ No field on the axis.
- ❖ Field strongest here.
- ❖ Linear fields i.e. $B_y \propto x$, $B_x \propto y$
- ❖ Focuses in horizontal plane.
- ❖ Defocuses in vertical plane.
- ❖ To have overall focusing the solution is to alternate the gradients.



Solution of the general equation of motion

The full mathematical expressions can be found in all reference books on the subject Here we note that:

- When $K > 0$ the motion is stable and sinusoidal
- When $K < 0$ the motion is unstable and hyperbolic
- When $K = 0$ the motion is linear in s

All these results can be written in the general form

$$\begin{bmatrix} z \\ z' \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} z \\ z' \end{bmatrix}$$

where z can be x or y

Note that the moduli of all these matrices will be (and must be) unity!
This condition conserves phase space and provides a useful check

What's next?

Today's HEP accelerators are nearing practical limits. What can be done?

- 1982 ECFA held the first workshop of a series on advanced accelerating techniques; “Challenge of Ultra-high Energies”, in Oxford (UK). The goal was a new acceleration technique capable of reaching PeV energies with equipment of a practical size

Essential ingredients are:

- A new acceleration mechanism, transverse and longitudinal (phase) stability, stability against collective effects

The candidates were:

- Plasma accelerator, Wake fields accelerator, Lasers on dense bunches, but the search is still on

How far is beyond?

- The CERN LHC operates at 2x8 TeV, only cosmic rays can show what lies beyond
- The cosmic ray spectrum is expected to extend up to the **Planck energy** ($1.22 \cdot 10^{28}$ eV, **about 10^{15} times higher than the LHC!**)
- The Planck energy is energy expected for a vibrating string theory and is roughly 2 billion Joules

Conclusions

- Using only very basic principles of classical electromagnetism and relativity (and first order approximations) we were able to describe the elementary mechanisms particle accelerators are based on to work
- The material covered in this seminar does amount only less than 1% of the full subject of Accelerator Physics (and still not Accelerator Science)!
- We did not exploit very important topics, such as the remaining transverse dynamics (Hill's equation, Twiss functions...), longitudinal dynamics (emittance, RF buckets, momentum compaction...), higher order magnetic lenses (sextupoles, octupoles), chromaticity, collective effects, instabilities, luminosity (fundamental concept for colliders!)...
- However the aim was just to instill the curiosity (but not sure to have reached the goal) about a very fascinating and rapidly expanding subject, applying very fundamental laws of Physics
- Some key events were mentioned in order to clarify how many efforts are still nowadays put on Accelerator Physics research

- Lee, “Accelerator Physics”
- Wiedemann, “Particle accelerator Physics”
- Wille, “The Physics of particle accelerators”
- Bryant, “Juas lectures”

And many many others...

Thanks for your kind attention