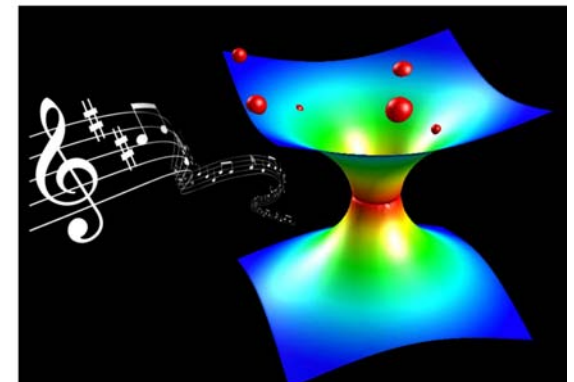
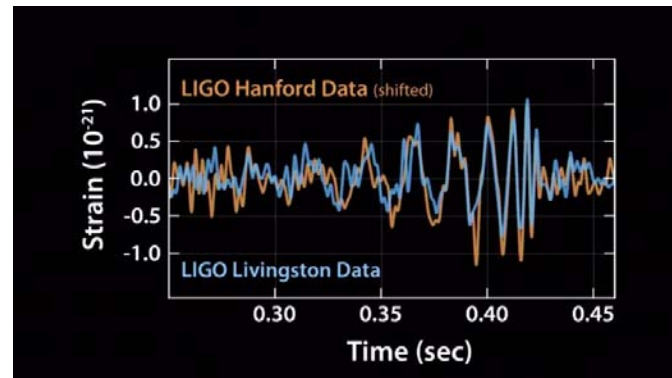
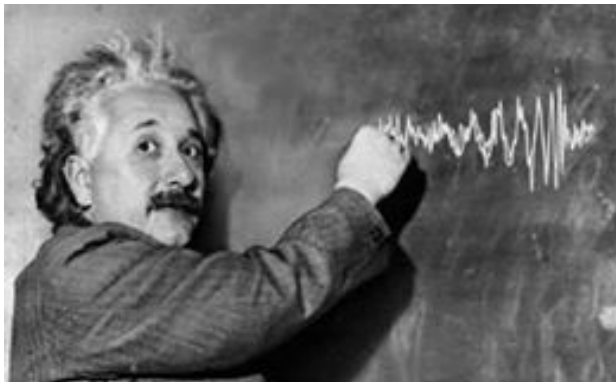


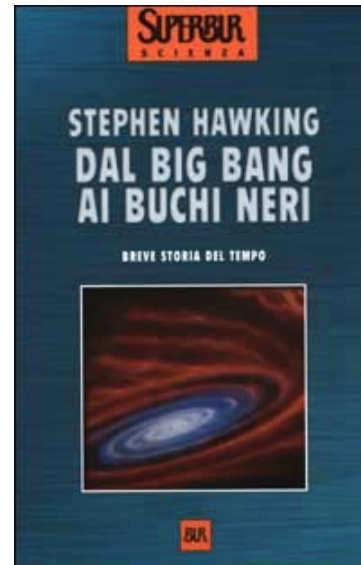
Onde gravitazionali: *la musica dello spaziotempo*



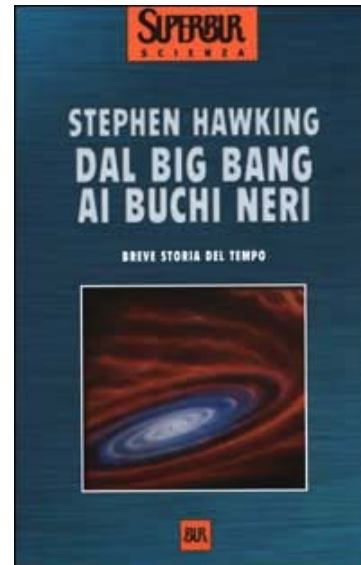
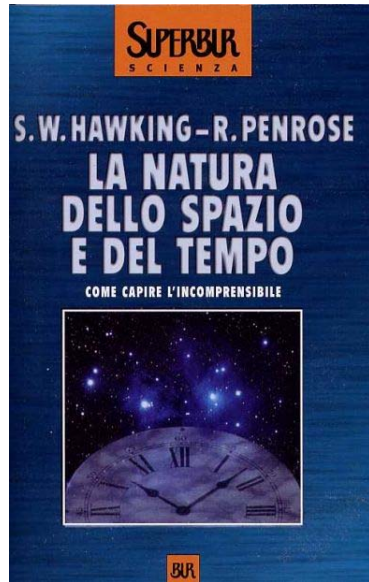
Paolo Pani

Università Roma Sapienza & INFN Roma1

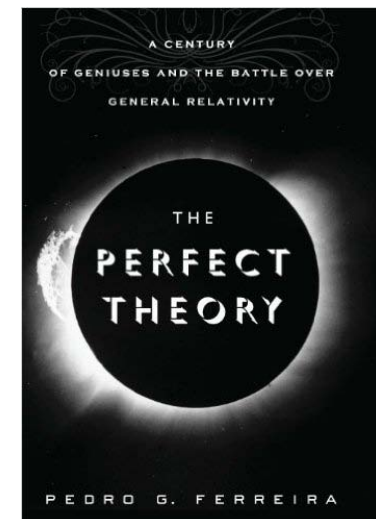
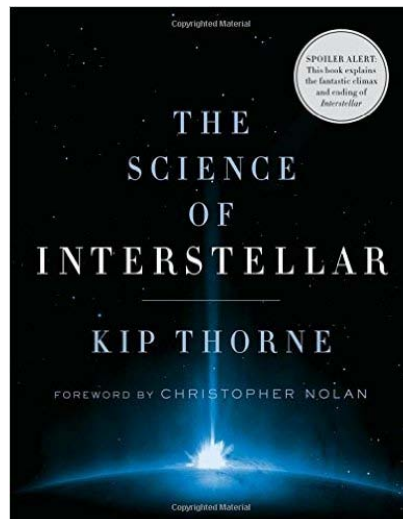
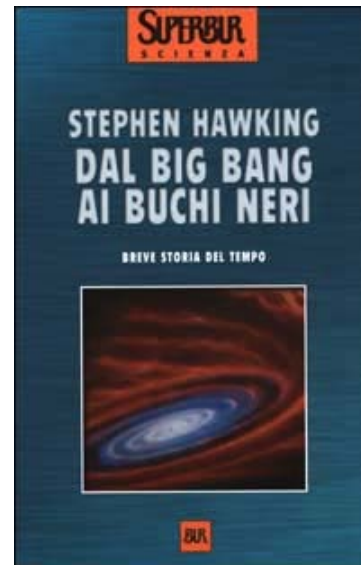
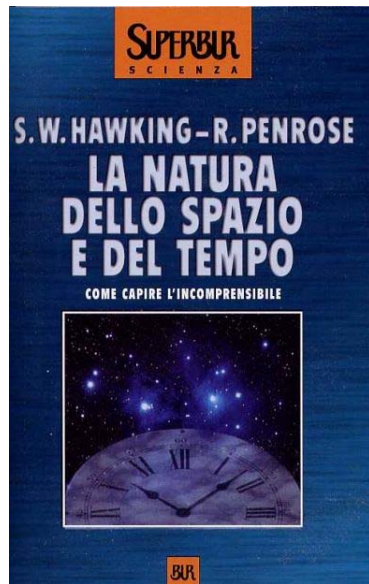
Da dove cominciare



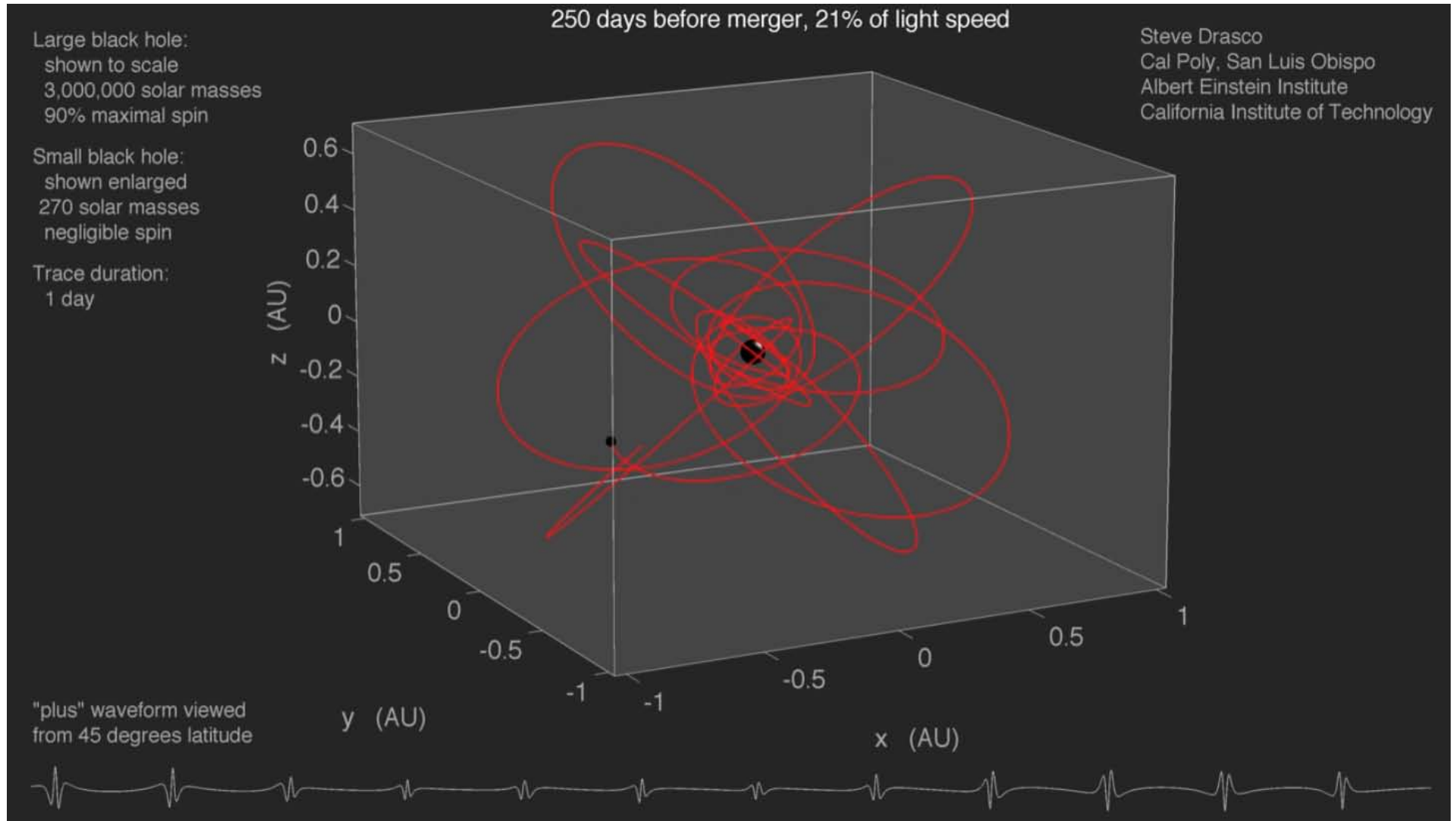
Da dove cominciare



Da dove cominciare



“Ascoltare” un'onda gravitazionale



Perché gli oggetti cadono?

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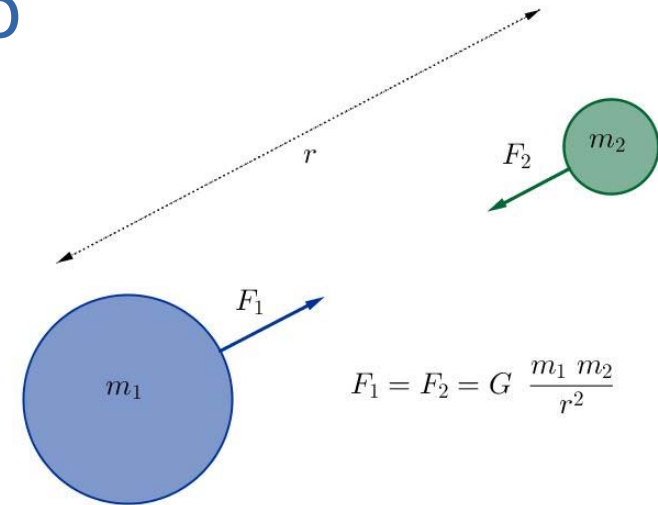
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- Newton / Galileo → forze a distanza

$$\vec{F} = m^{(i)} \vec{a} \quad \vec{F} = G \frac{m_1^{(g)} m_2^{(g)}}{r^2} \hat{r}$$

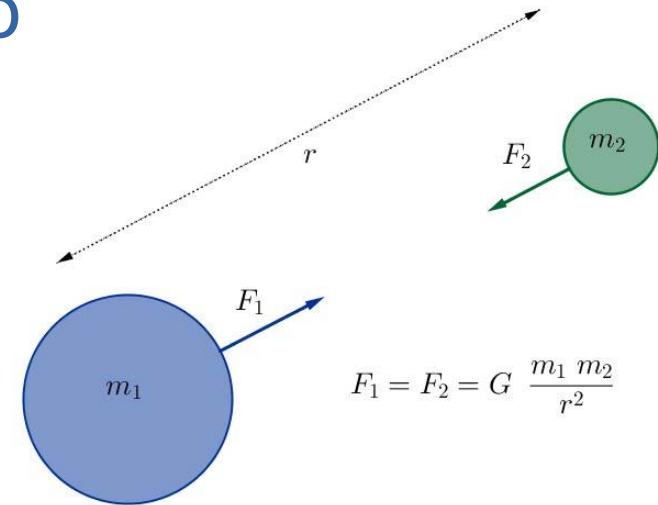


- Einstein → curvatura spaziotempo
-

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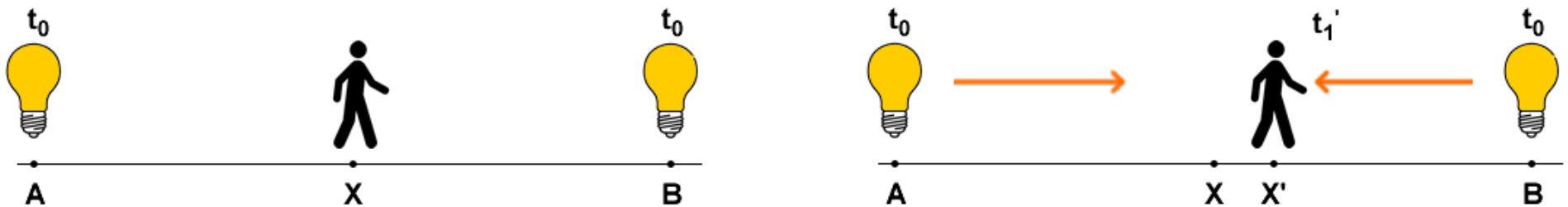


- Einstein → curvatura spaziotempo

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Il tour-de-force di Einstein

- ♦ **1905**: “annus mirabilis” → Relatività speciale
 - Principio di relatività speciale (sistemi inerziali)
 - Limite della velocità della luce → costante
 - No interazioni istantanee a distanza



- ♦ 1908: Principio di equivalenza $m^{(i)} = m^{(g)}$
 - ♦ **1915**: Relatività generale (“gravitodinamica”)
-

Il tour-de-force di Einstein

- ♦ **1915**: Relatività generale (“gravitodinamica”)
- ♦ Principio di equivalenza:

E' impossibile distinguere **localmente** un campo gravitazionale dalle forze d'inerzia che appaiono in un riferimento non inerziale



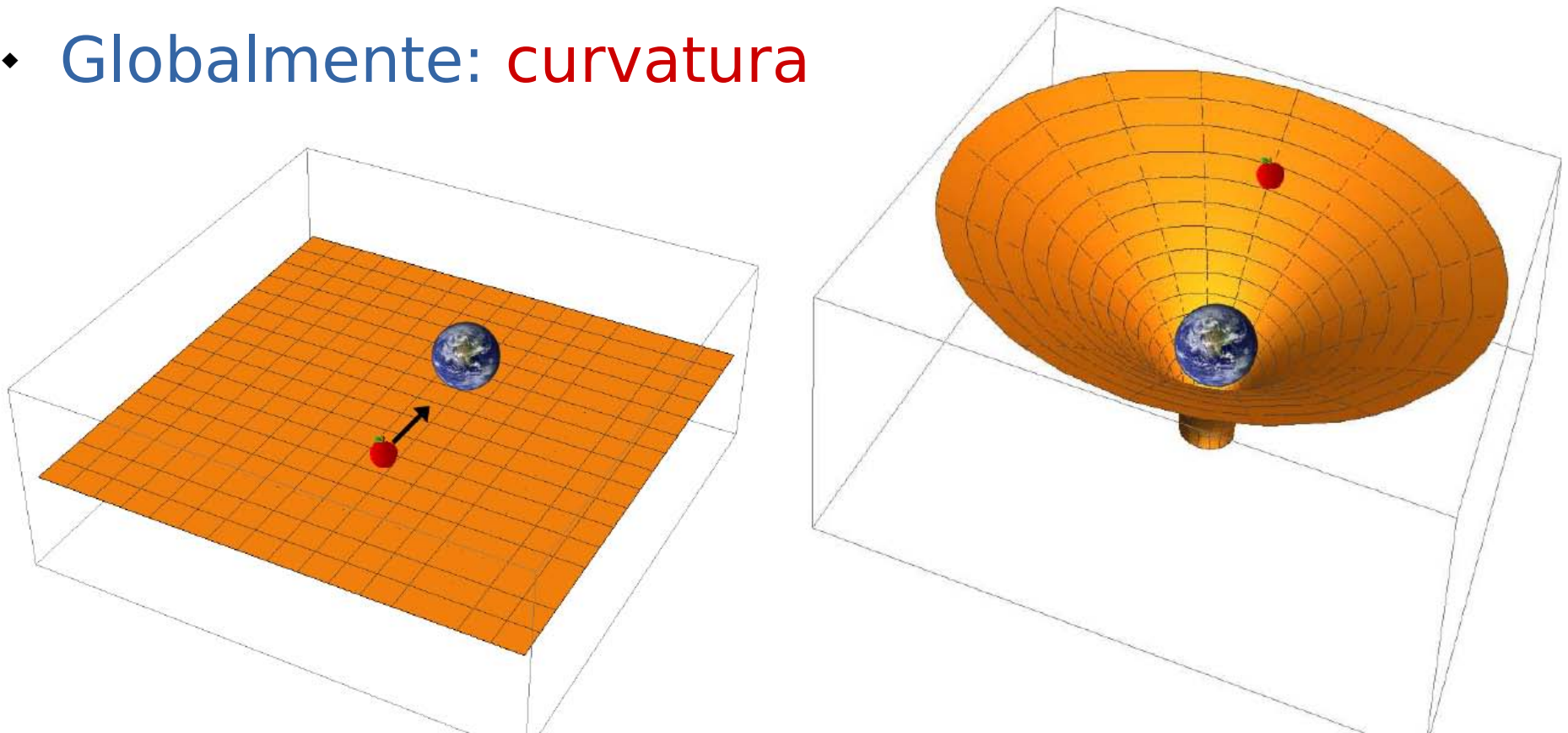
Il tour-de-force di Einstein

- ◆ Localmente: spaziotempo di **Minkowski**

(distanza spaziotempo)² = $-c^2 \text{tempo}^2 + \text{distanza}^2$

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

- ◆ Globalmente: **curvatura**



Spaziotempo in relatività generale



$$ds^2 = -c^2 g_{00} dt^2 + g_{11} dx^2 + g_{22} dy^2 + g_{33} dz^2$$

Curvatura

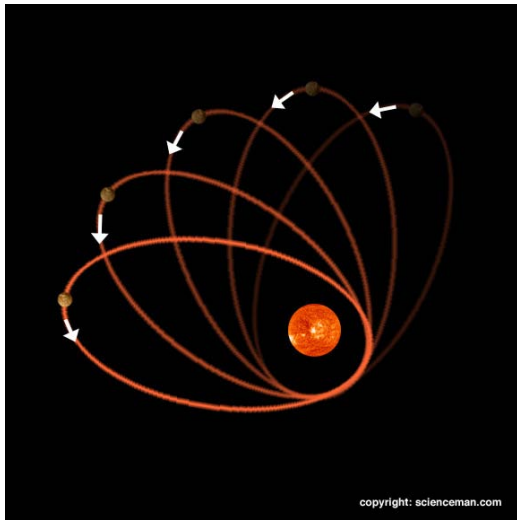
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Materia/energia

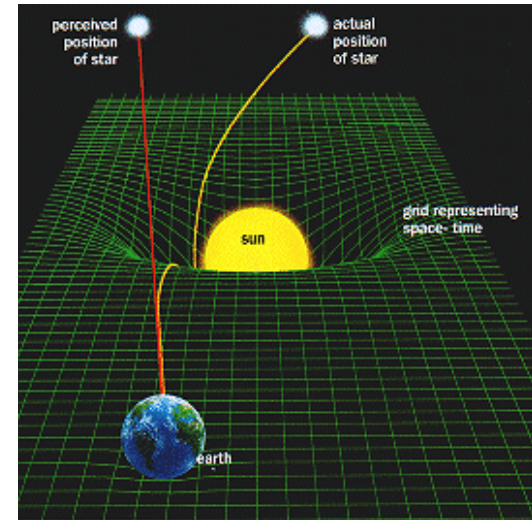
“Lo spaziotempo dice alla materia come muoversi,
e la materia dice allo spaziotempo come incurvarsi”

Ma quindi Newton?

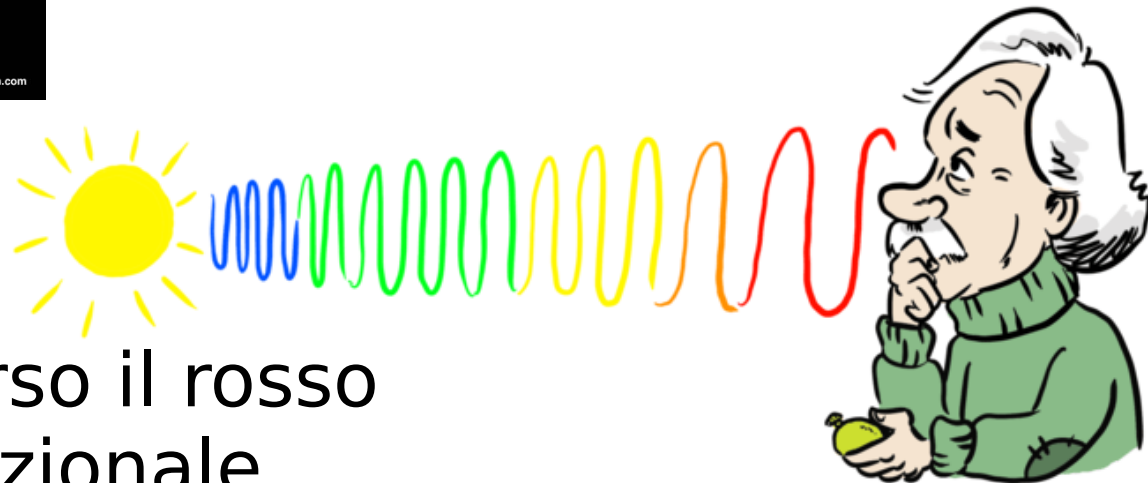
- ◆ Precessione del perielio



- ◆ Deflessione della luce

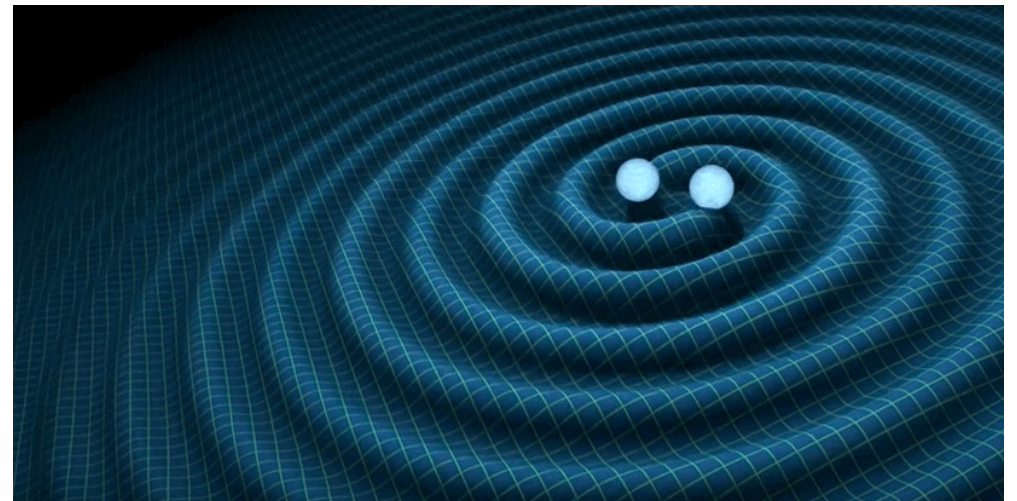
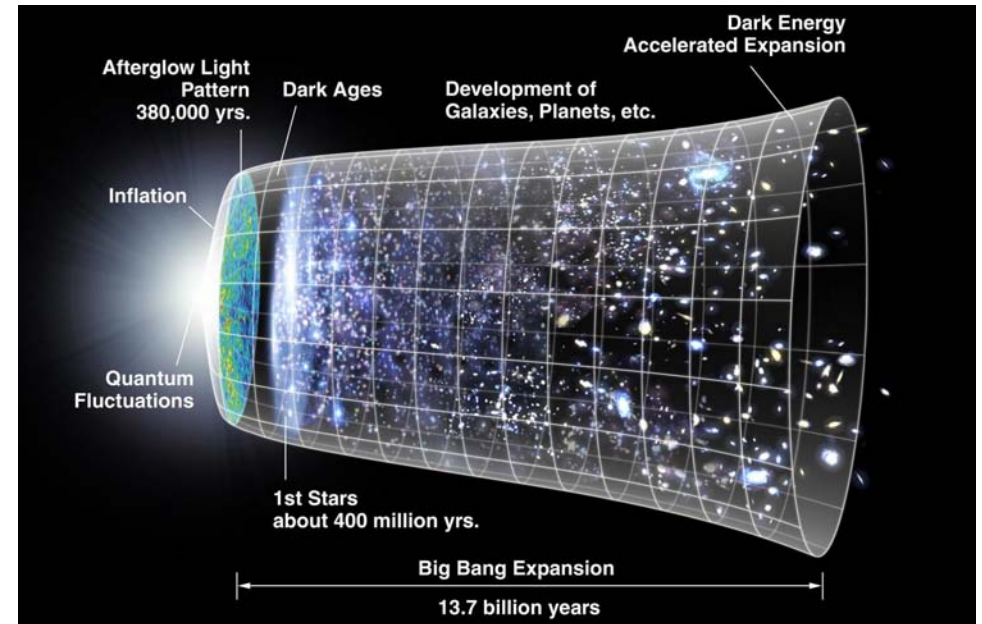


- ◆ Spostamento verso il rosso (redshift) gravitazionale



Hic Sunt Leones

- ◆ Cosmologia
- ◆ Buchi neri
- ◆ Onde gravitazionali



Breve teoria delle onde gravitazionali

- Linearizzazione delle equazioni di Einstein

$$g_{ab} = \eta_{ab} + h_{ab} + \mathcal{O}(h^2)$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} \rightarrow \begin{cases} \square \bar{h}_{ab} = -\frac{16\pi G}{c^4}T_{ab} \\ \nabla_a \bar{h}_b^a = 0 \end{cases}$$



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Nel vuoto

$$\frac{\partial^2 h_{ab}}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 h_{ab}}{\partial t^2} = 0$$

“Incrispature” dello spaziotempo che si propagano alla velocità della luce

$$h_{ab} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & h_+ & h_\times \\ 0 & 0 & h_\times & -h_+ \end{pmatrix}$$

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- Approx di quadrupolo**

- Campo debole
- Velocita' non relativistiche

$$\bar{h}_{ij}(t, \vec{x}) = \frac{2G}{c^4} \frac{1}{r} \frac{d^2}{dt^2} [q_{ij}(t - r/c)]$$

~ momento di quadrupolo

Nel vuoto

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~ momento di quadrupolo

Nel vuoto

$$\frac{\partial^2 h_{ab}}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 h_{ab}}{\partial t^2} = 0$$

"Incespature" dello spaziotempo che si propagano alla velocita' della luce

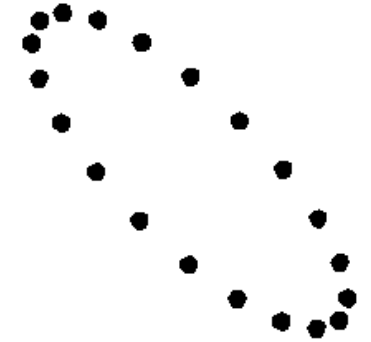
$$h_{ab} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & h_+ & h_\times \\ 0 & 0 & h_\times & -h_+ \end{pmatrix}$$

Per stelle binarie

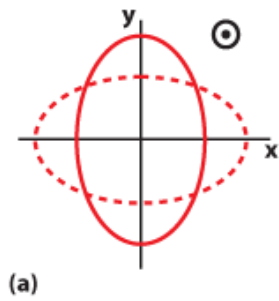
Polarizzazione onde gravitazionali



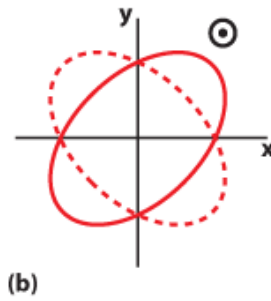
$$h_{ab} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & h_+ & h_\times \\ 0 & 0 & h_\times & -h_+ \end{pmatrix}$$



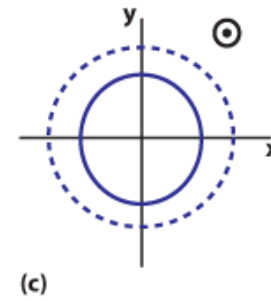
polarizzazione +



(a)

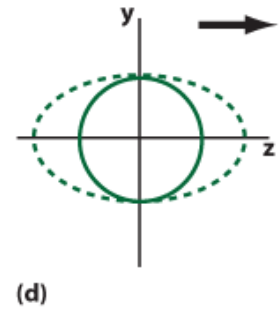


(b)

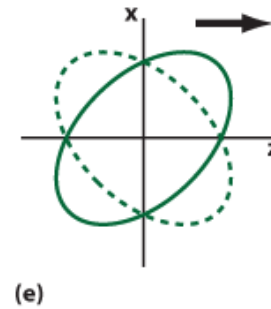


(c)

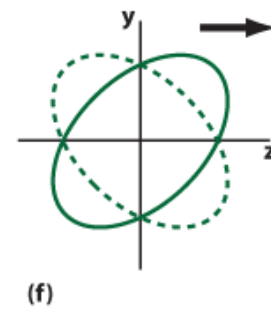
polarizzazione x



(d)



(e)



(f)

Onde gravitazionali VS Onde EM

Elettromagnetismo

Oscillazioni di **E** e **B** *nello* spaziotempo

Lunghezza d'onda \ll dimensioni (img)

Emissione incoerente (antenna)

Dipolare

Interagisce con la materia

massa nulla – spin = 1

Velocità = c

Relatività Generale

Oscillazioni *dello* spaziotempo

Lunghezza d'onda \gtrsim dimensioni

Emissione coerente (chitarra)

Quadrupolare

Interagisce *debolmente* con la materia

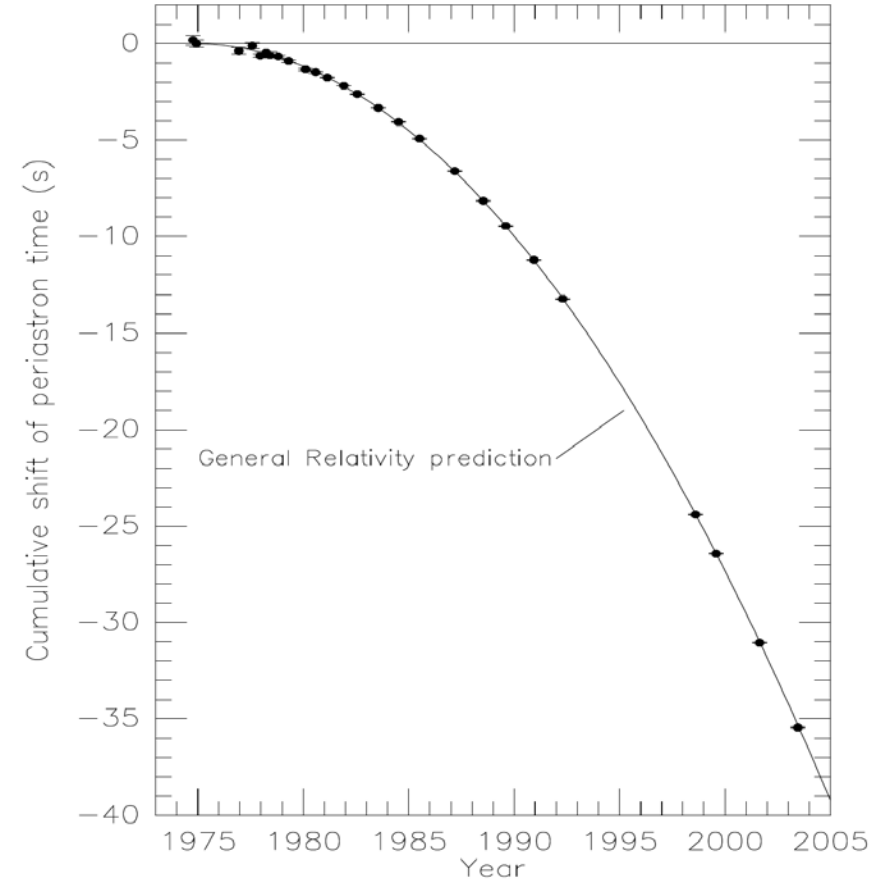
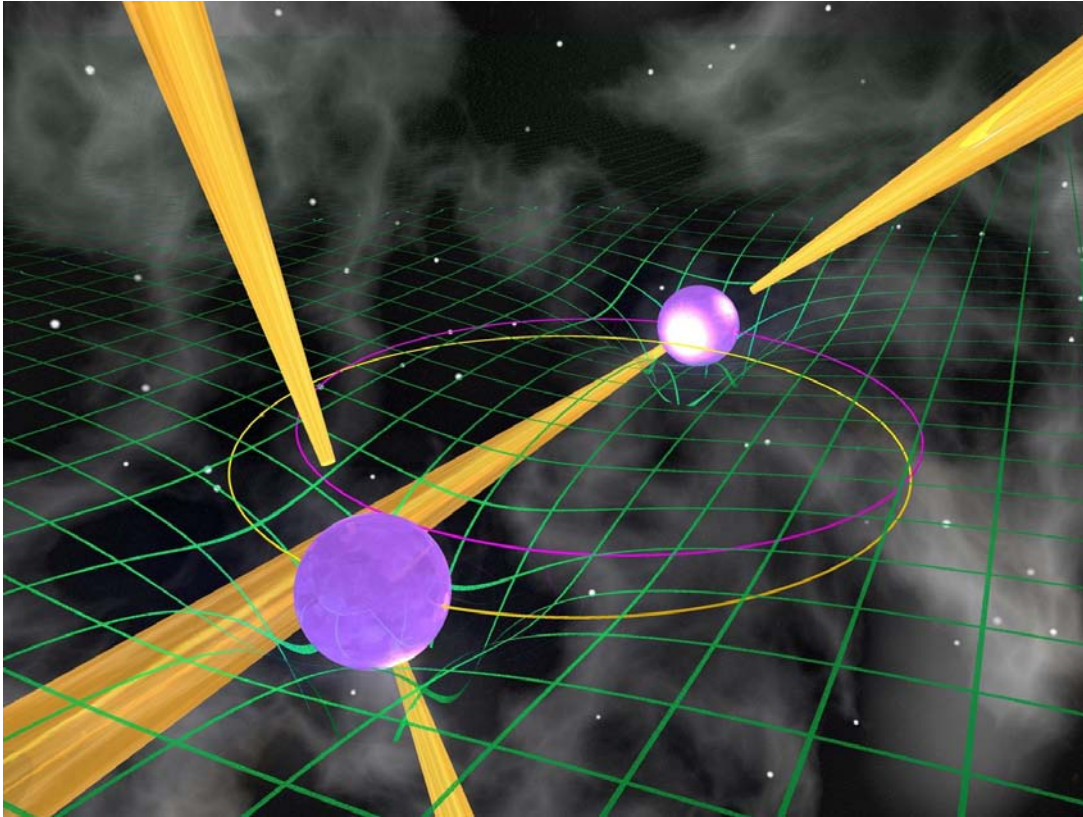
massa nulla – spin = 2

Velocità = c

Prima di LIGO: rivelazione indiretta

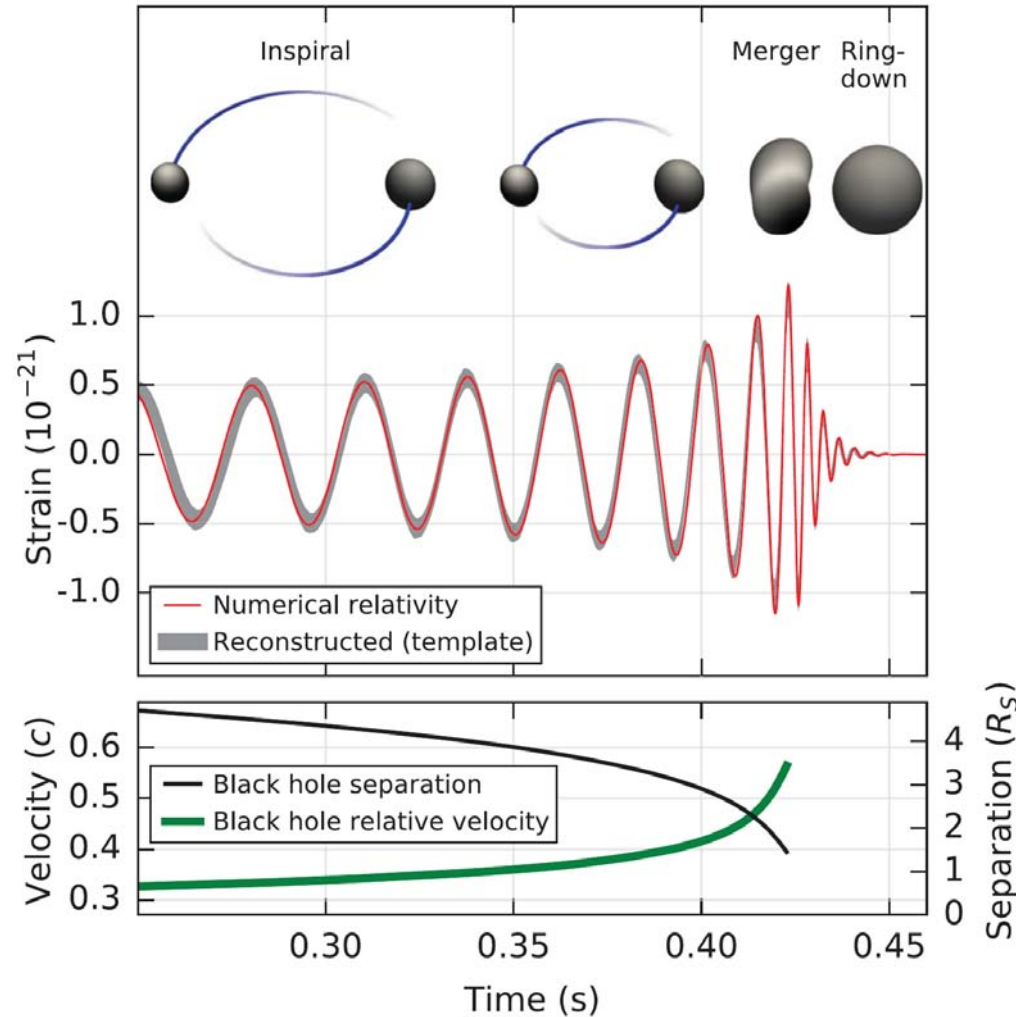
$$\frac{dE_{\text{GW}}}{dt} \sim \frac{32 G^4 M^5}{5 c^5 r_0^5}$$

Radio-pulsar binarie



Le 3 fasi della fusione gravitazionale

LIGO-Virgo Scientific Collaboration; Phys. Rev. Lett. 116, 061102



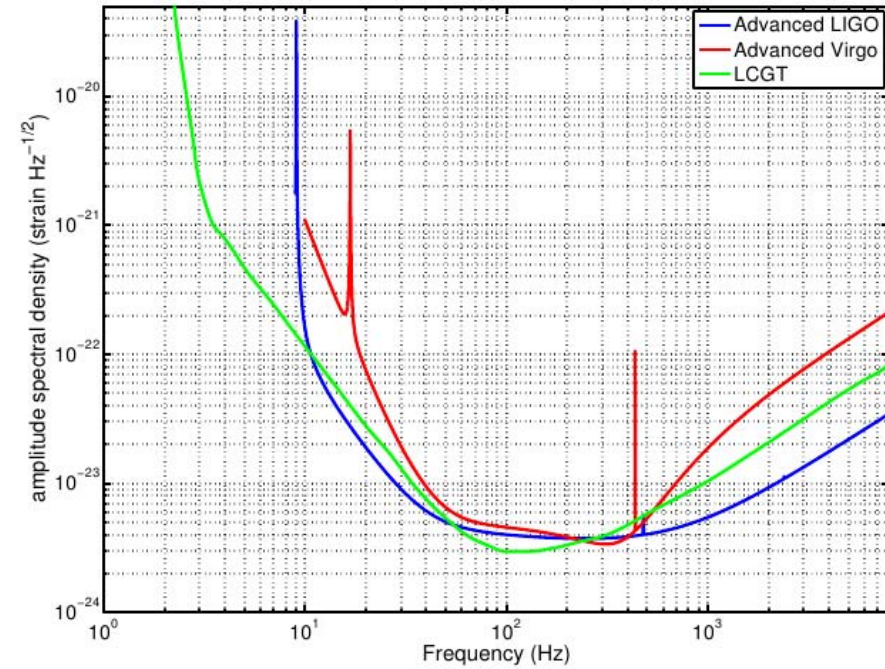
$$m_T \approx 62M_{\odot} < m_1 + m_2 \approx 65M_{\odot} \Rightarrow \Delta E \approx 3M_{\odot}c^2$$

Interferometri laser gravitazionali



Virgo @ Cascina (PI)

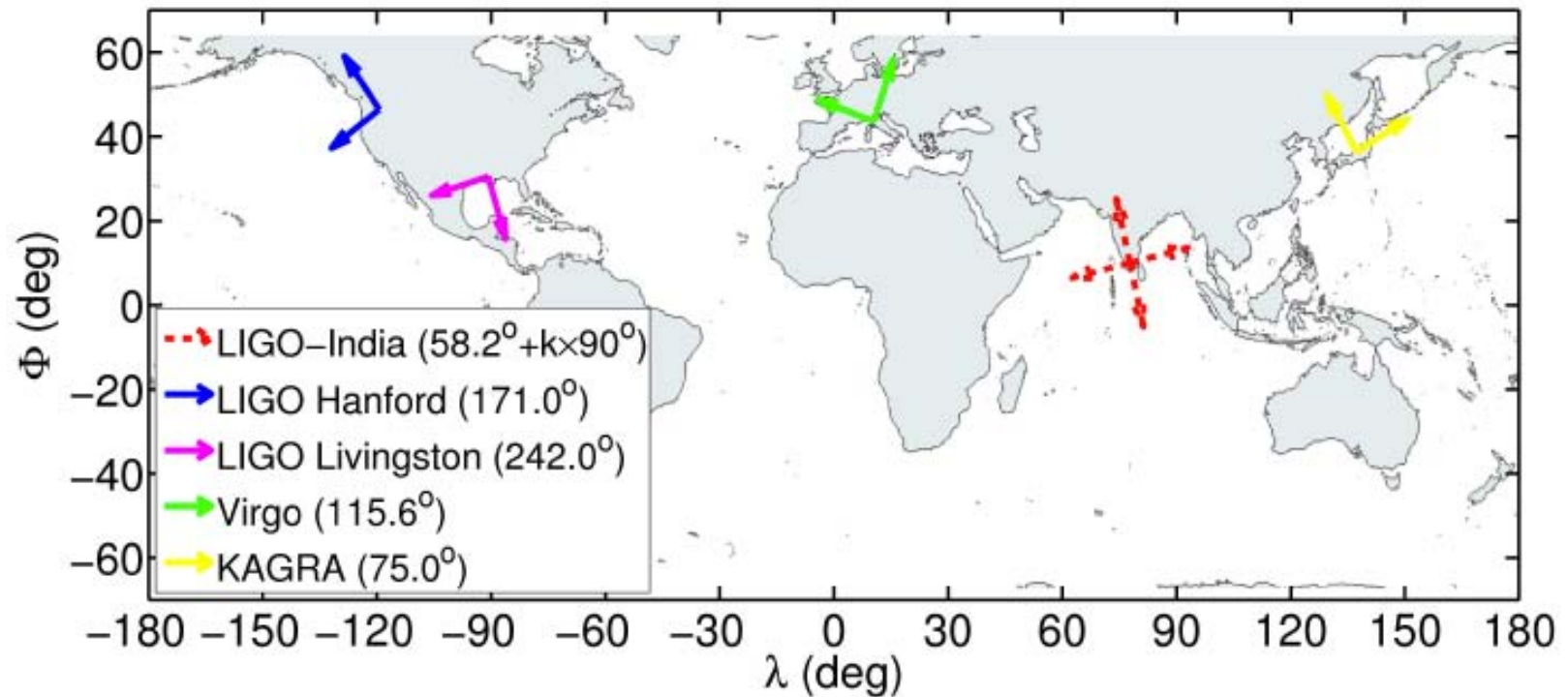
Sensitività



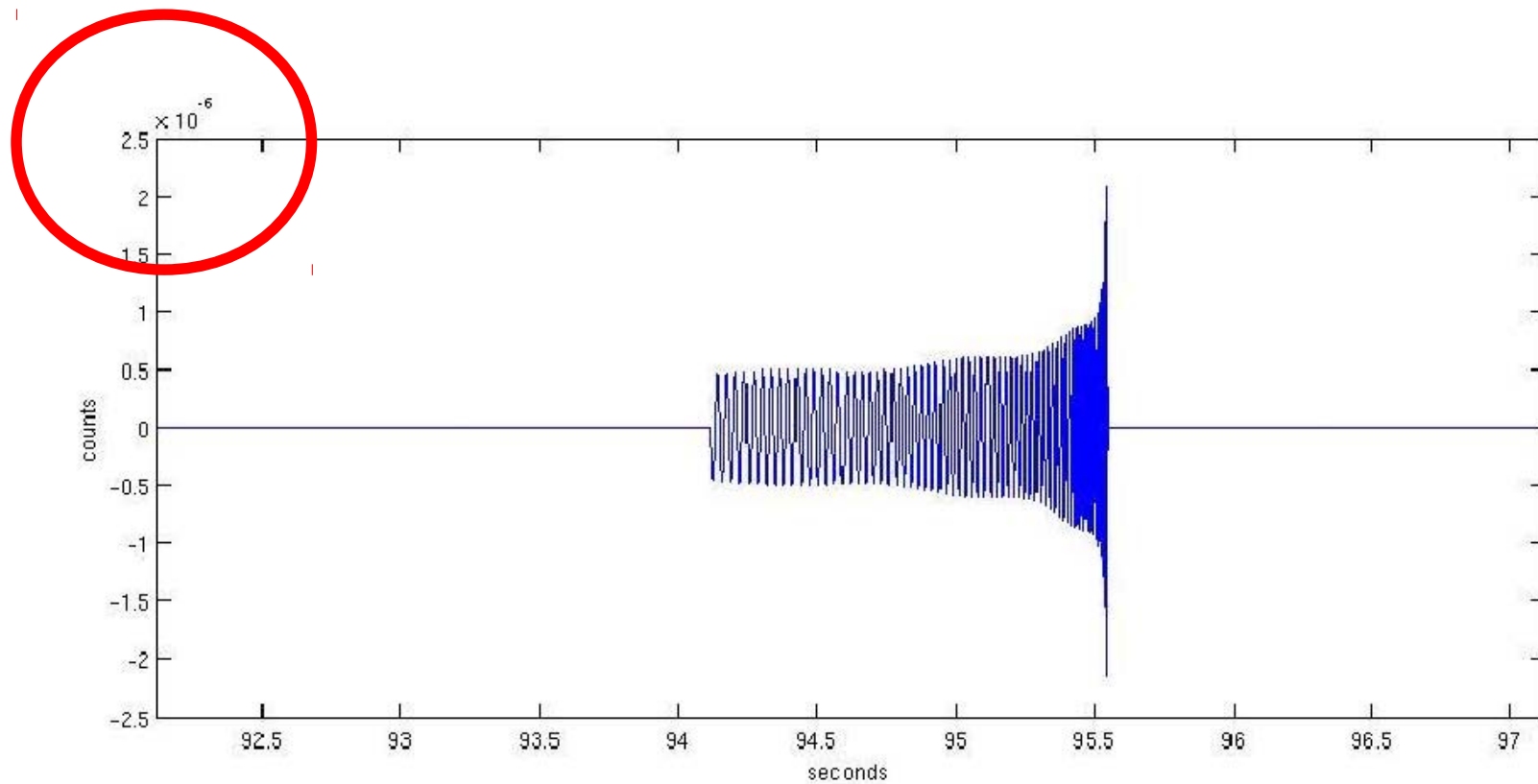
$$\delta L \sim 10^{-3} \frac{h}{10^{-21}} \frac{L}{\text{km}} \text{fm}$$

Sensibili a modifiche del raggio terrestre ~ dimensioni del protone!

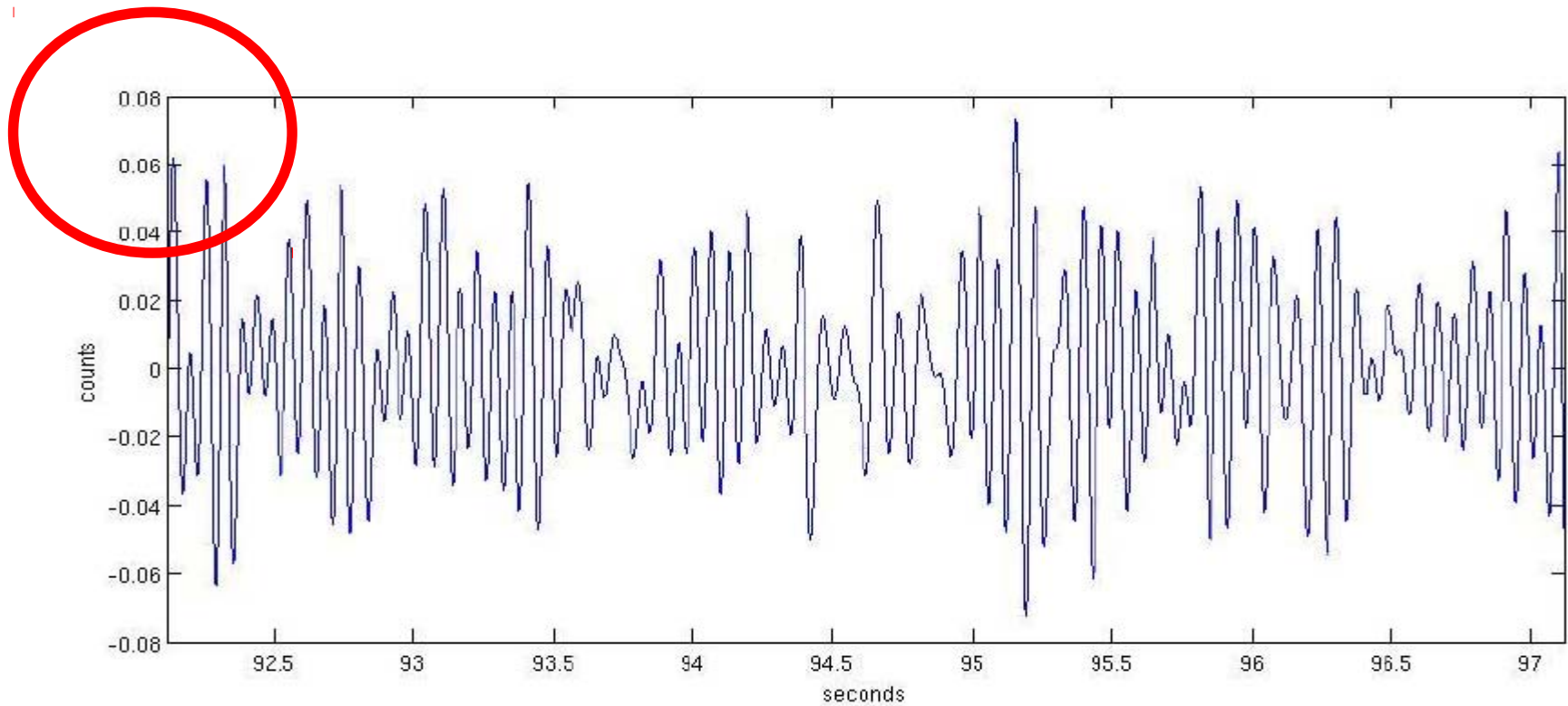
Network di interferometri



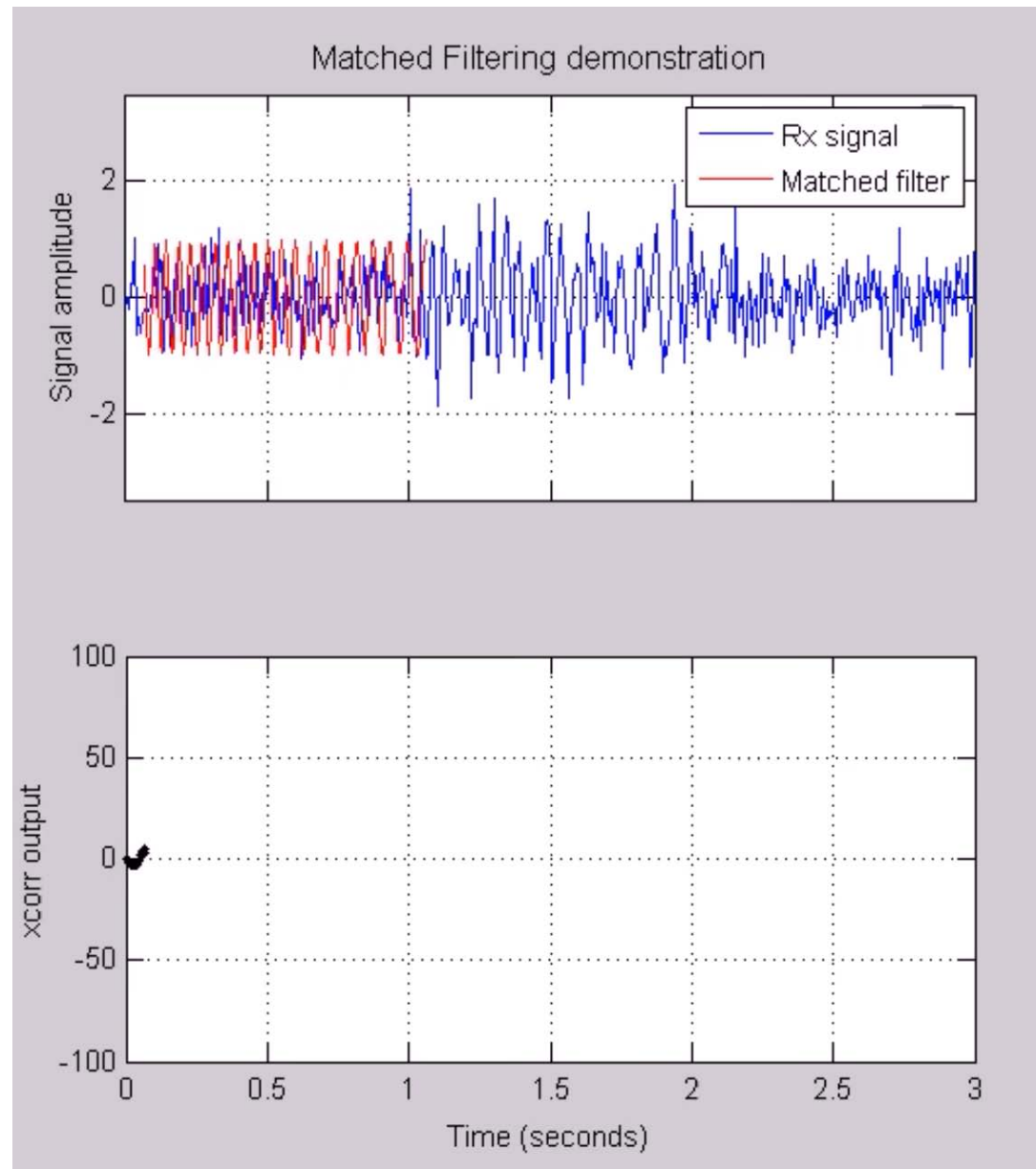
Cosa vorremmo osservare...



... e cosa invece osserviamo

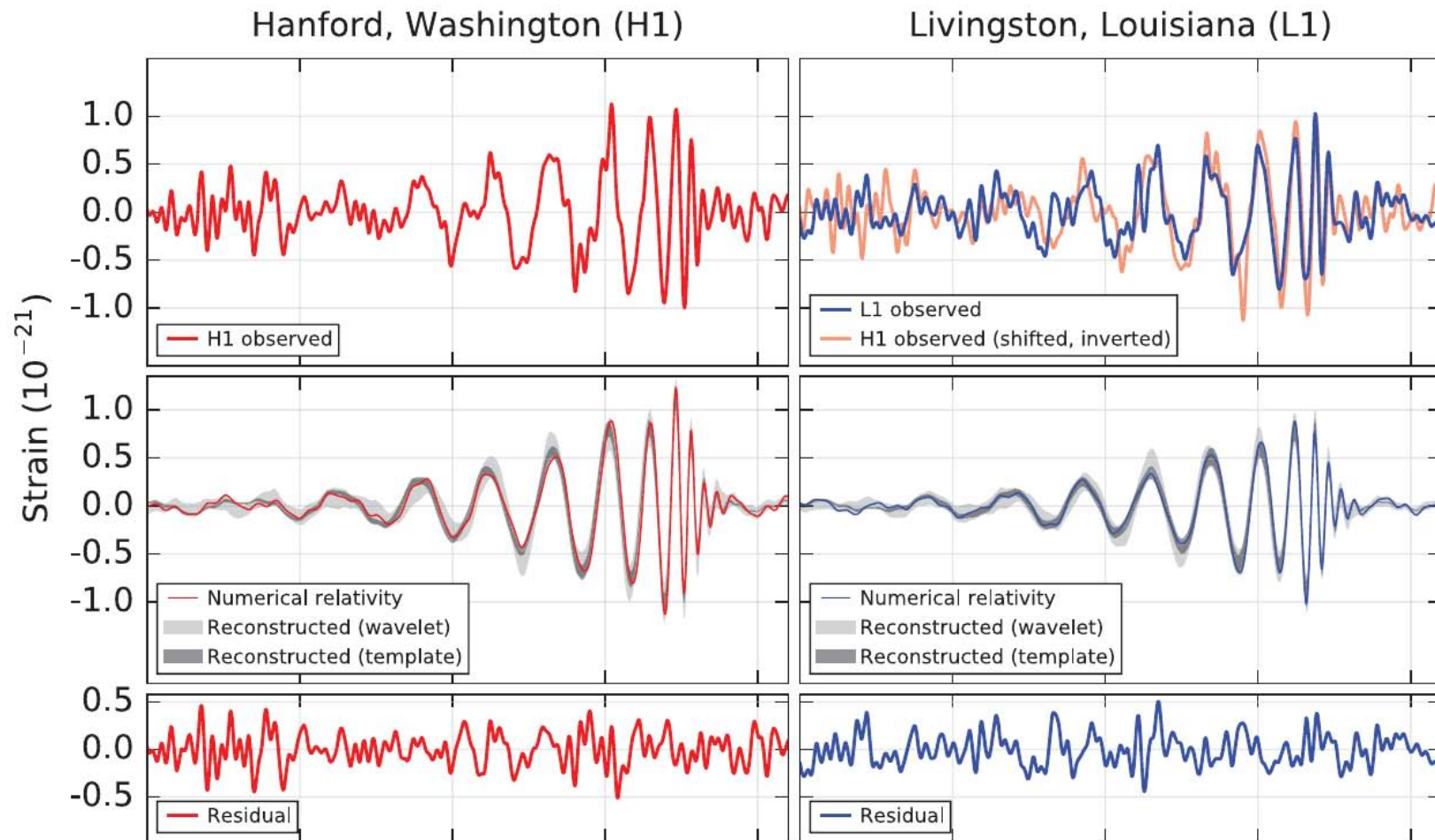


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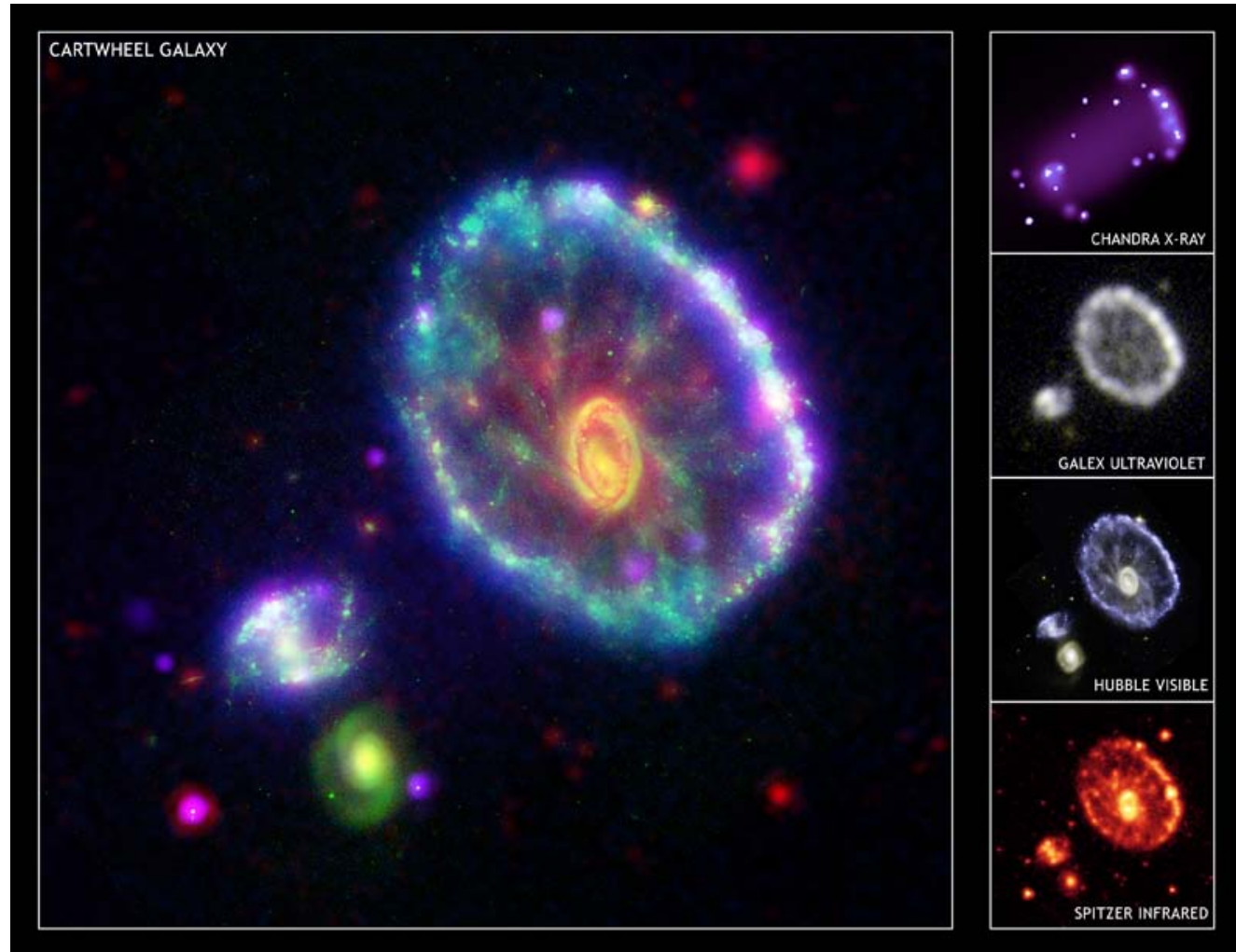


Il primo evento: GW150914

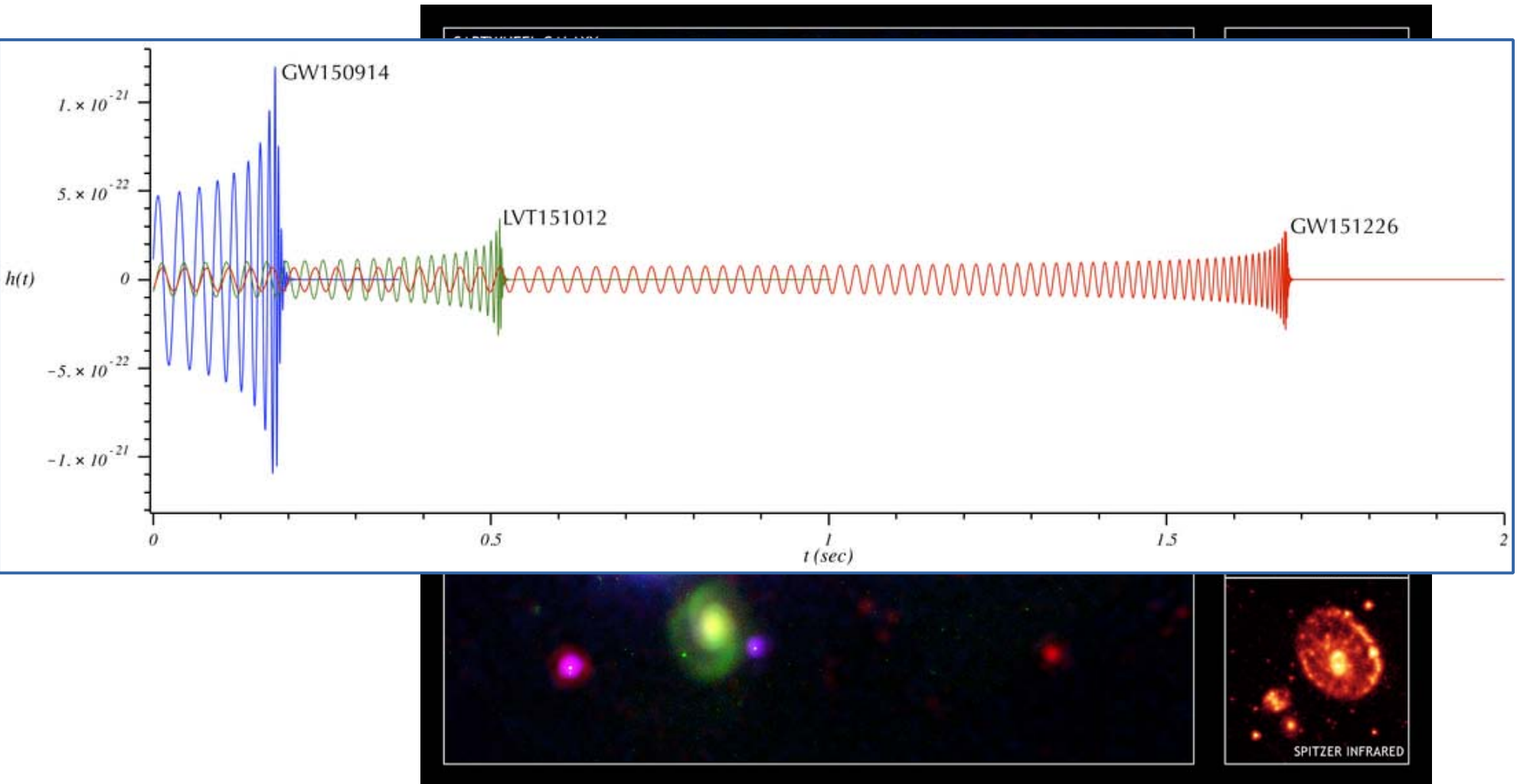
LIGO-Virgo Scientific Collaboration; Phys. Rev. Lett. 116, 061102



Una nuova finestra sull'universo



Una nuova finestra sull'universo



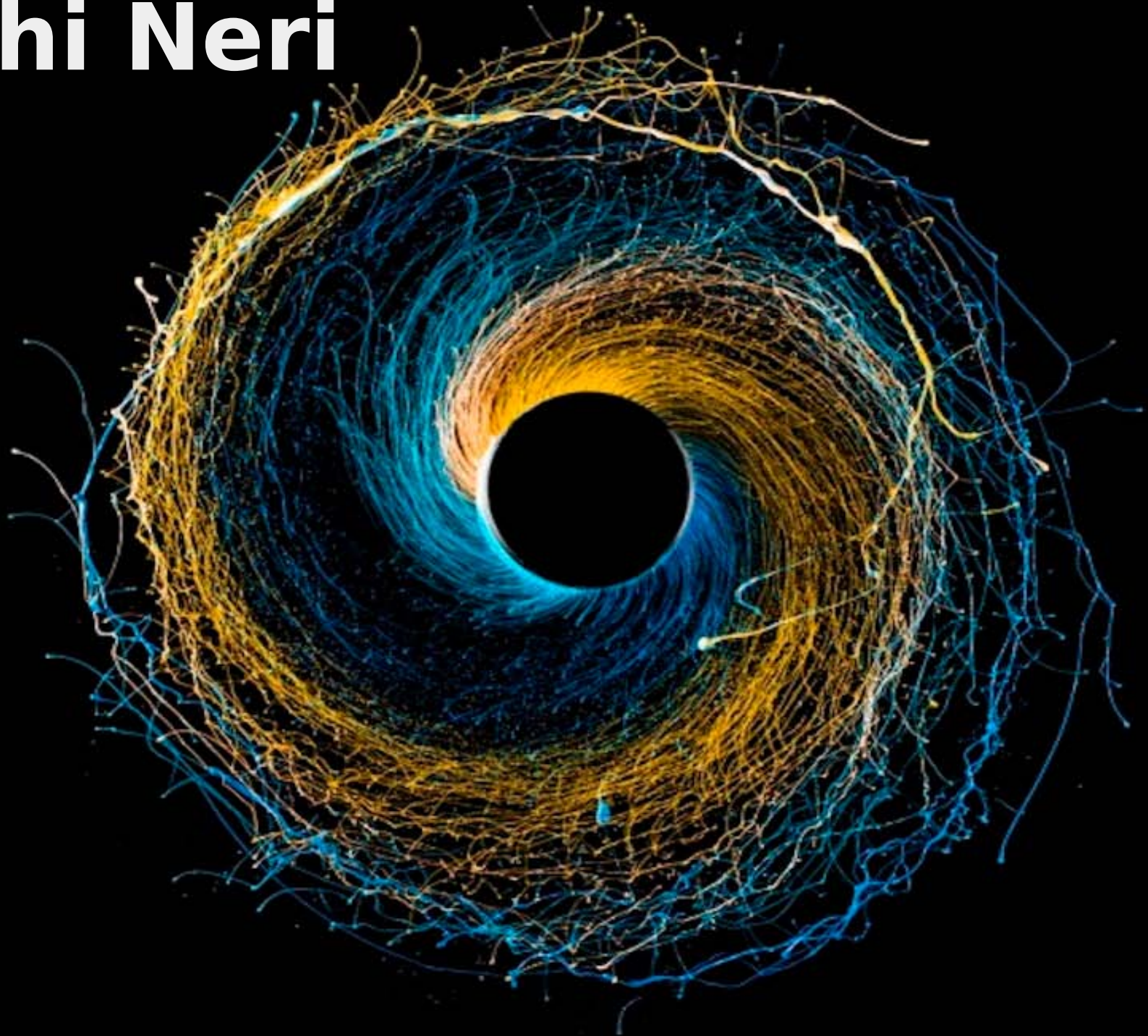
Einstein@home



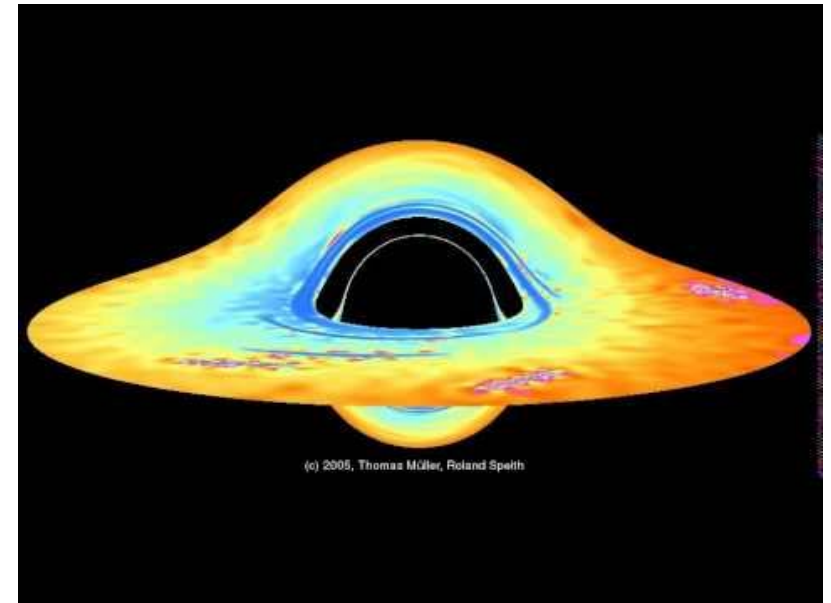
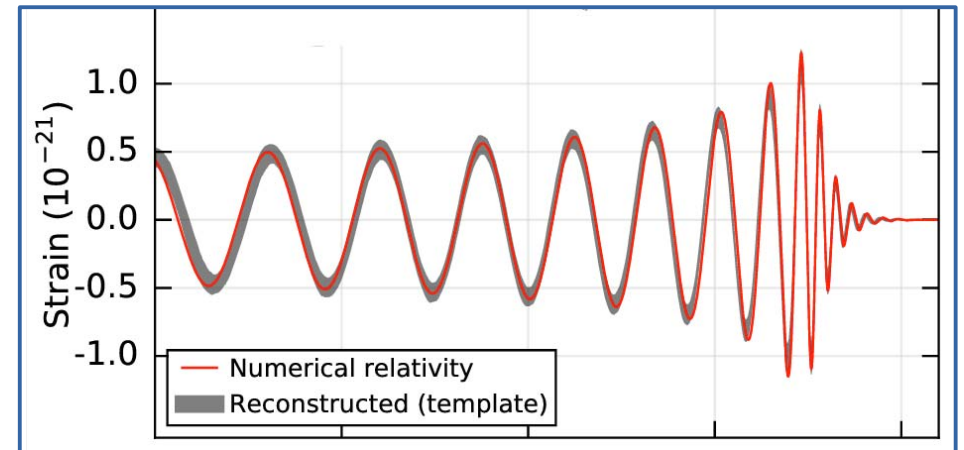
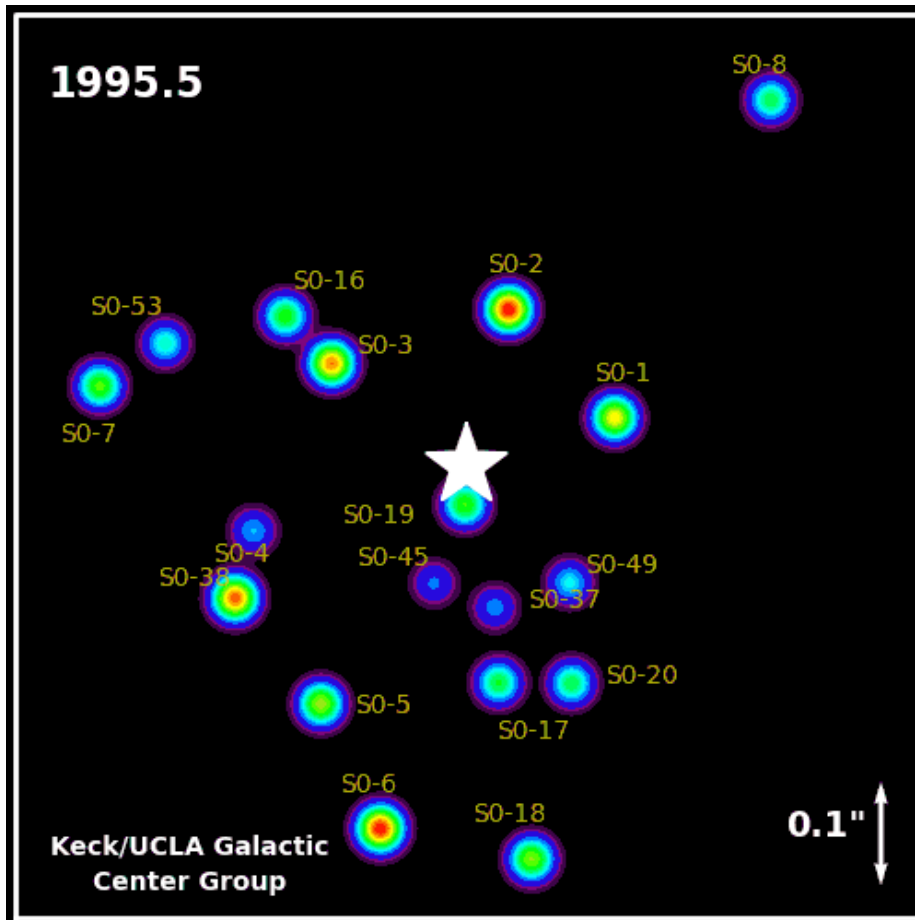
<http://einstein.phys.uwm.edu>



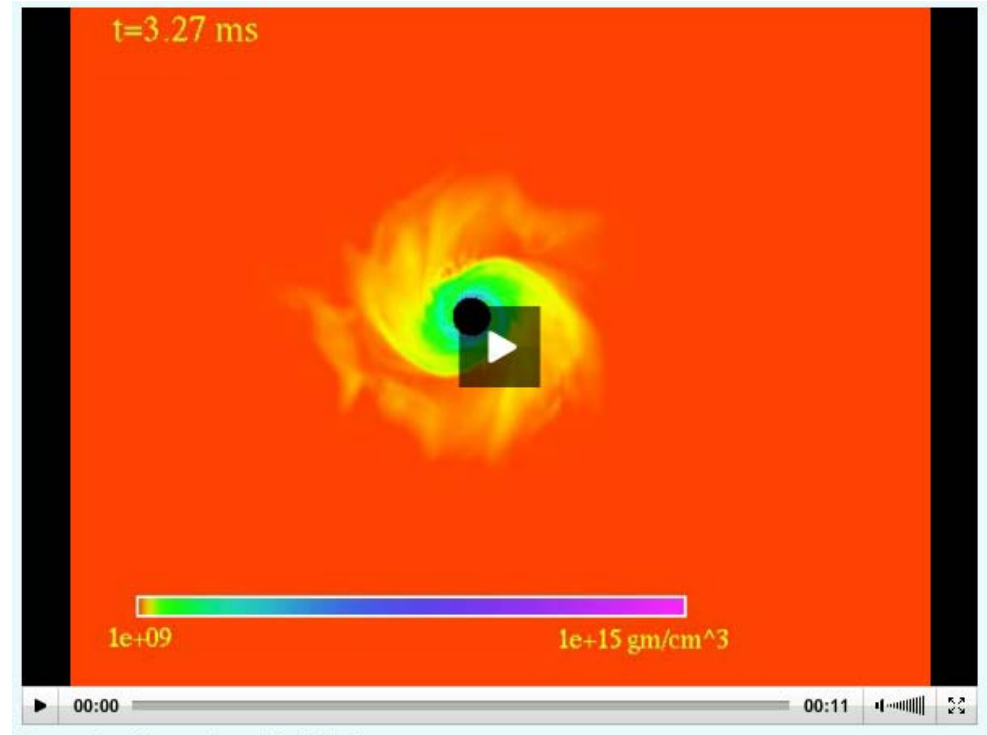
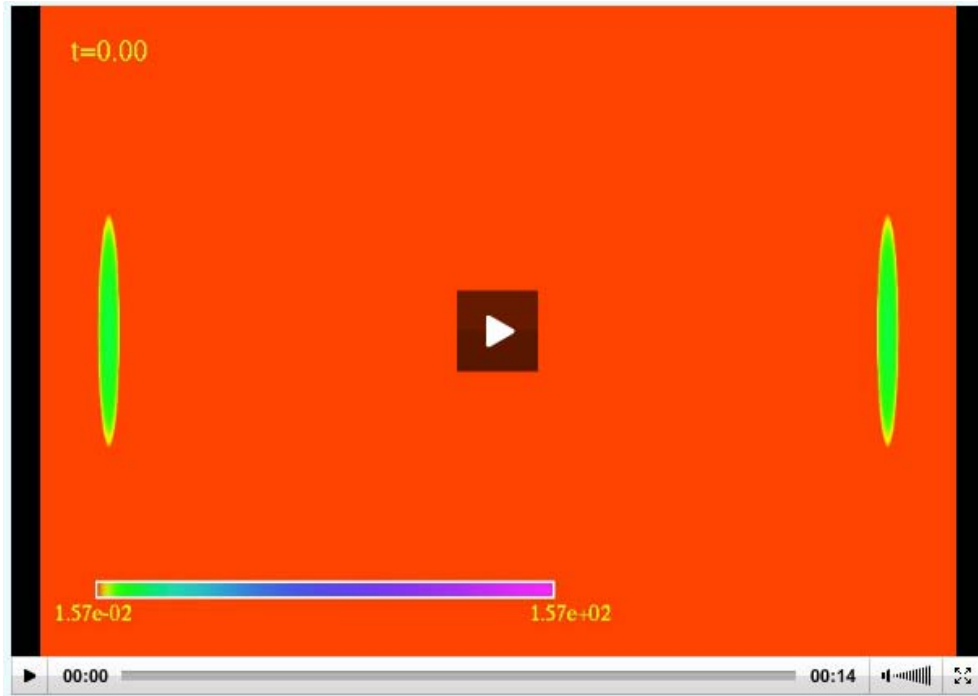
Buchi Neri



Buchi neri?



Come si forma un buco nero?



Credits: F. Pretorius (Princeton)

La soluzione di Schwarzschild

$$ds^2 = -c^2 \left(1 - \frac{R_S}{r}\right) dt^2 + \frac{1}{1 - \frac{R_S}{r}} dr^2 + r^2 d\Omega^2$$

Fattore di curvatura

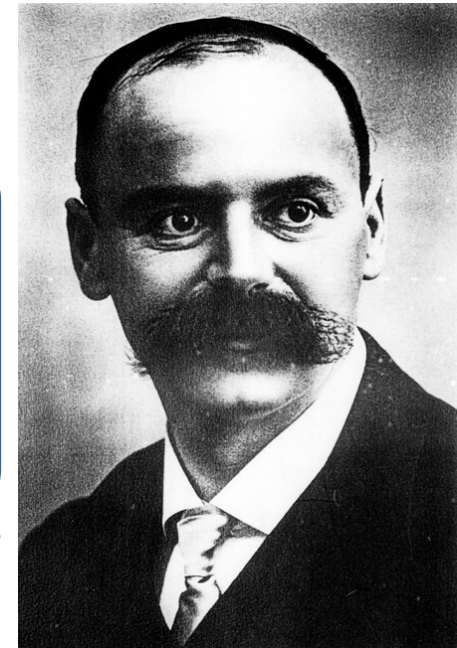
Sfera



“Teorema” dell'hula hoop

$$R_S = \frac{2GM}{c^2}$$

Raggio di Schwarzschild
(1916)



Il raggio di Schwarzschild

- Stella compatta → forte campo gravitazionale

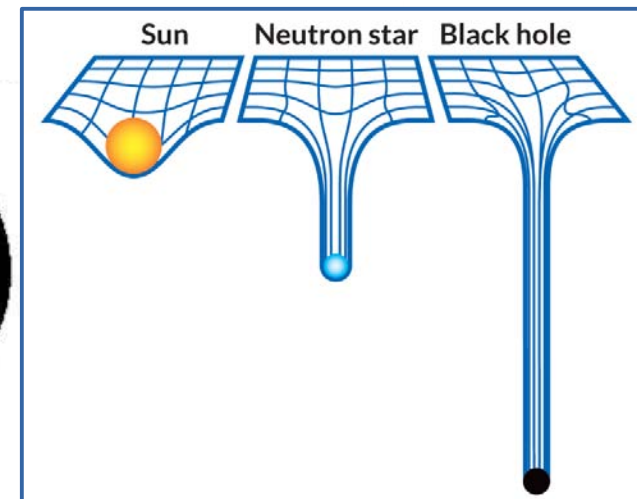
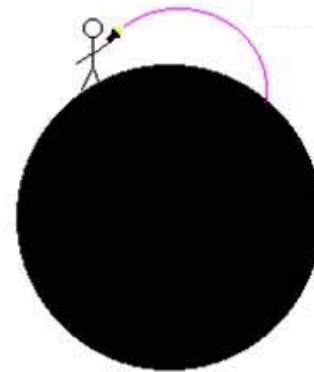
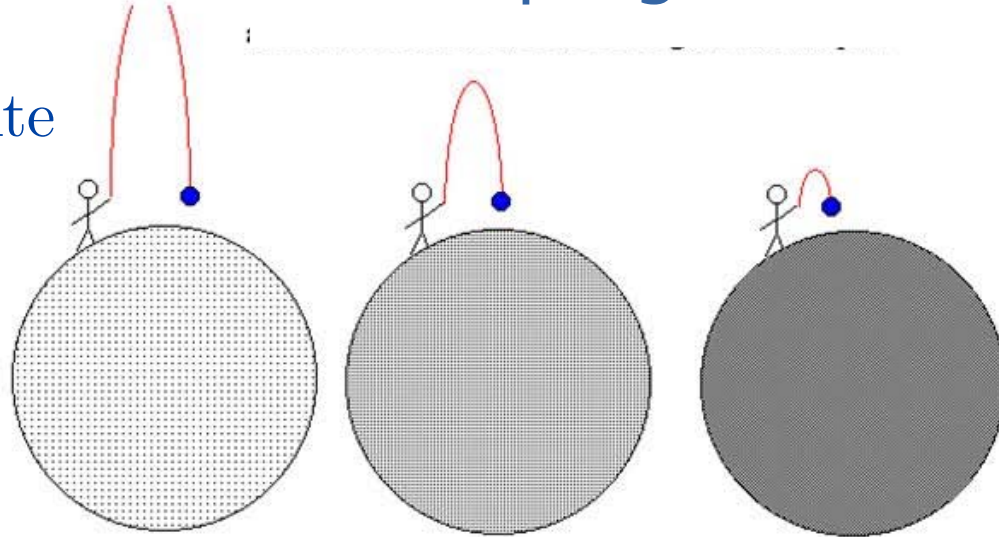
$$E_{\text{cinetica}} + E_{\text{pot.}} = \text{costante}$$

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = 0$$



- Velocità di fuga:

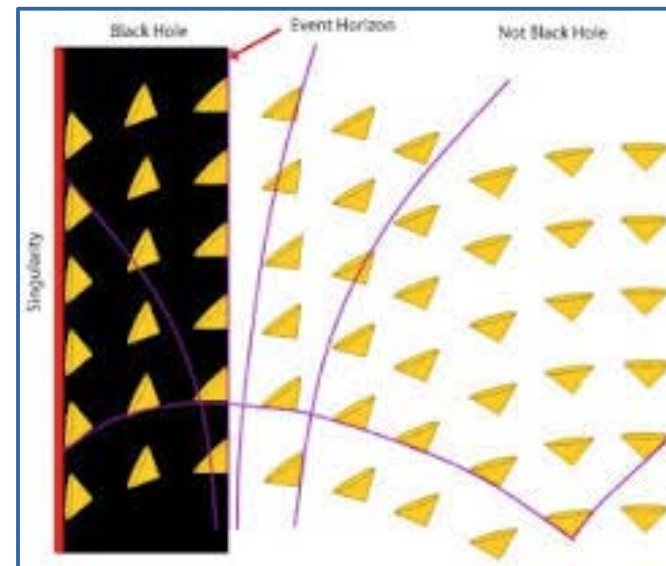
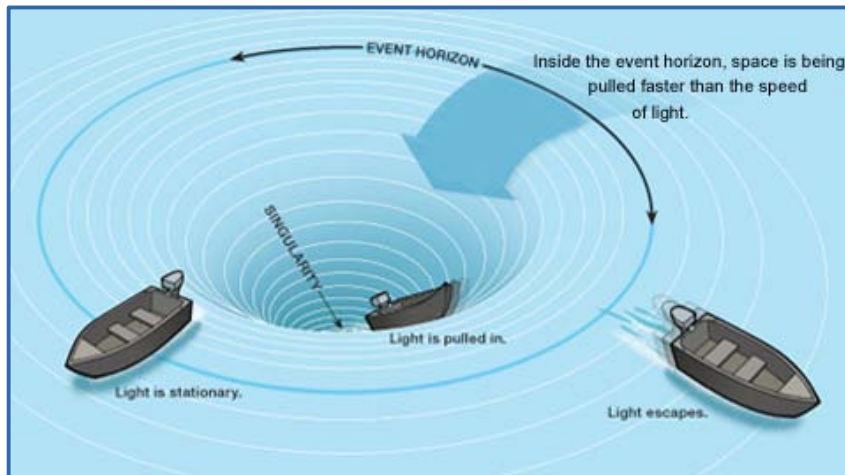
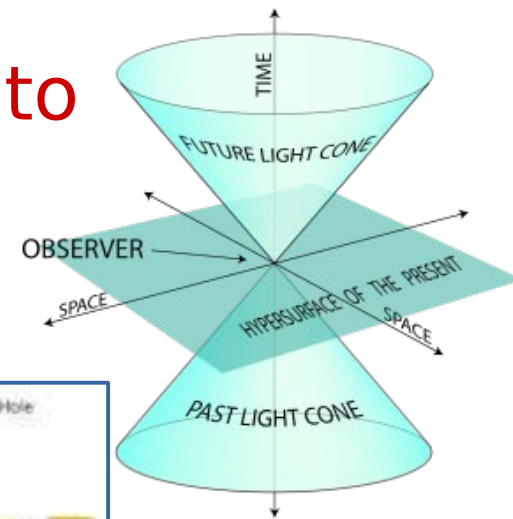
$$v_{\text{fuga}} = \sqrt{\frac{2GM}{R_S}} = c \Rightarrow R_S = \frac{2GM}{c^2}$$



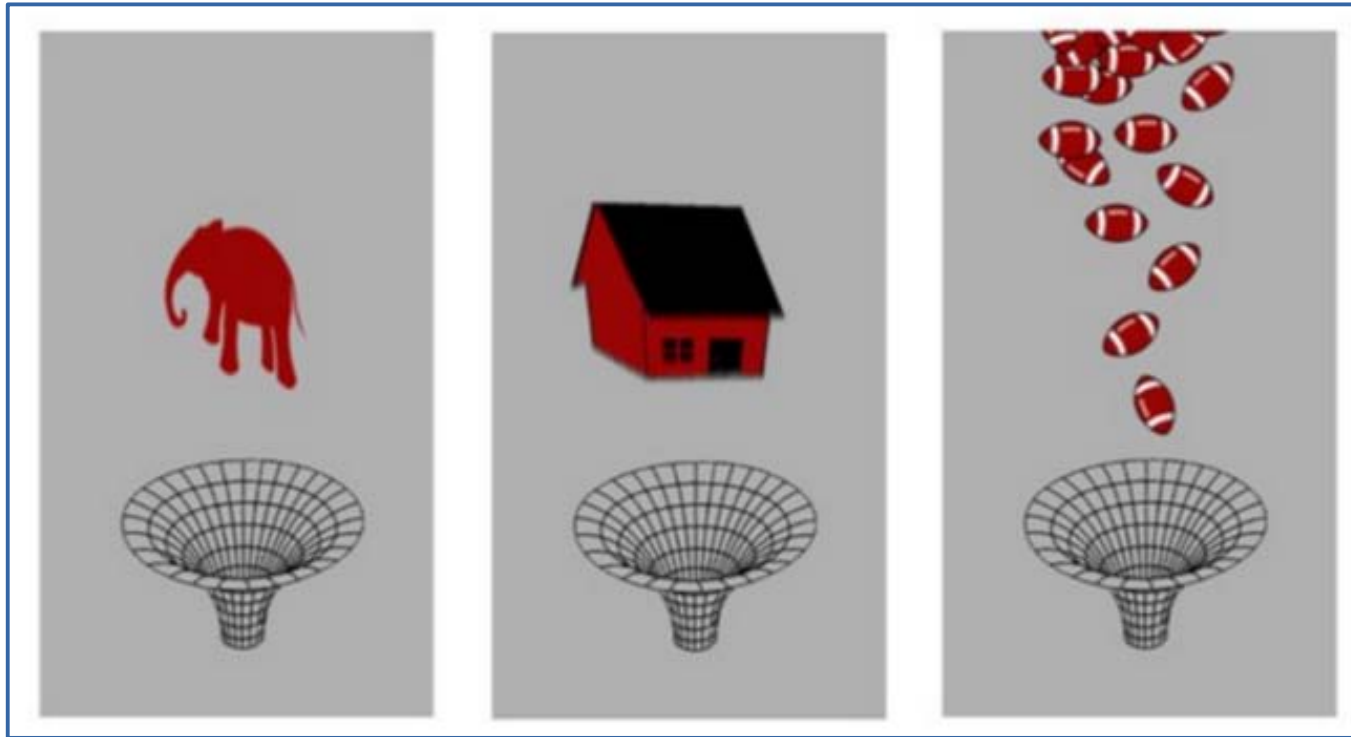
Singularità di Schwarzschild

$$ds^2 = -c^2 \left(1 - \frac{R_S}{r}\right) dt^2 + \frac{1}{1 - \frac{R_S}{r}} dr^2 + r^2 d\Omega^2$$

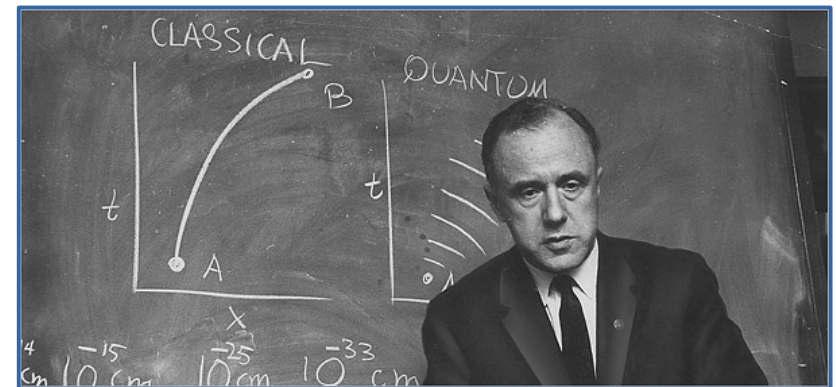
- Raggio di Schwarzschild → **redshift infinito**
- $r=0$ → **singularità di curvatura**



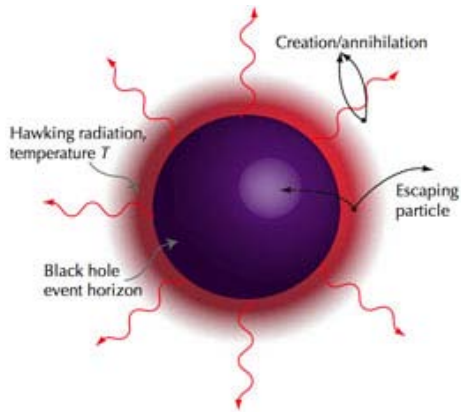
“Black holes have no hair”



I buchi neri sono identici
fra loro → contano solo la
massa e lo **spin**



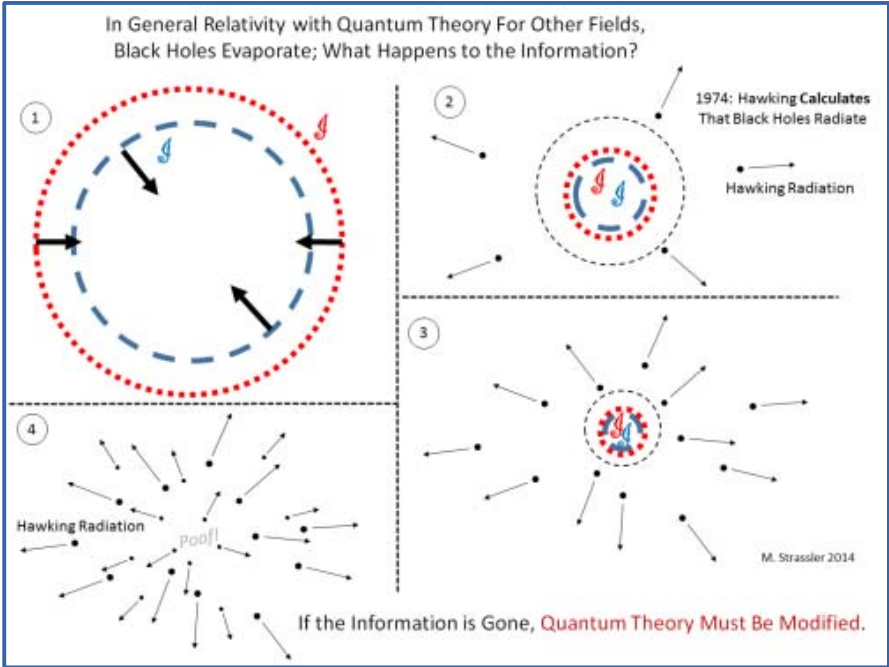
Evaporazione dei buchi neri



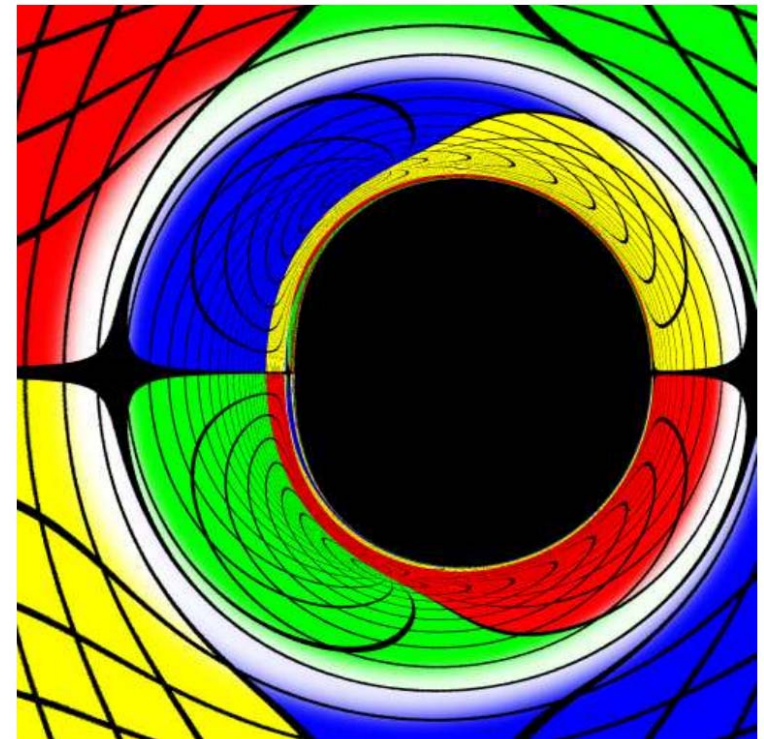
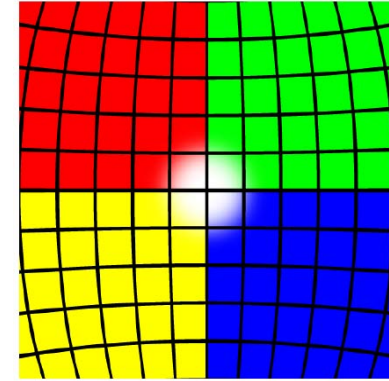
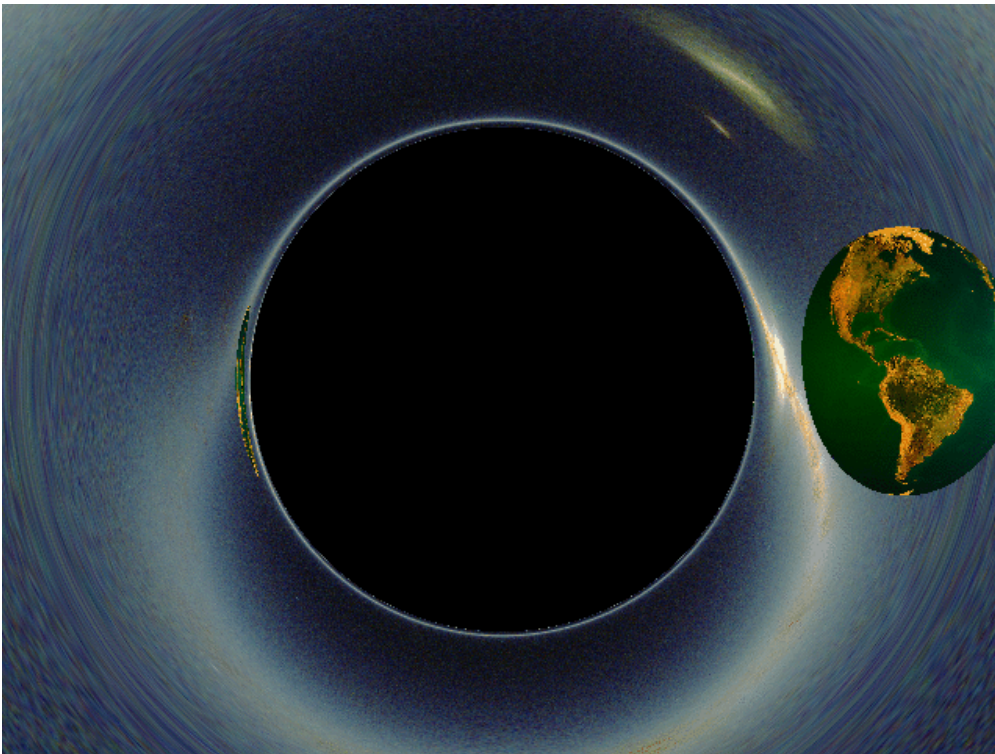
$$T_H = \frac{\hbar c^3}{8\pi G M k_B}$$



- ◆ Più bella formula della Fisica?
- ◆ **Paradosso dell'informazione**



La “silhouette” di un buco nero



Oltre Einstein?

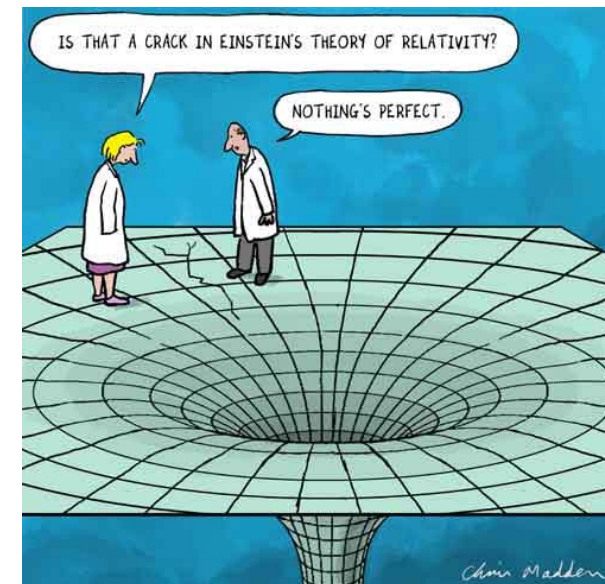
La Relatività Generale
non è rinormalizzabile

Materia Oscura

$$G_{\mu\nu} \stackrel{???}{\neq} T_{\mu\nu}$$

Regimi inesplorati

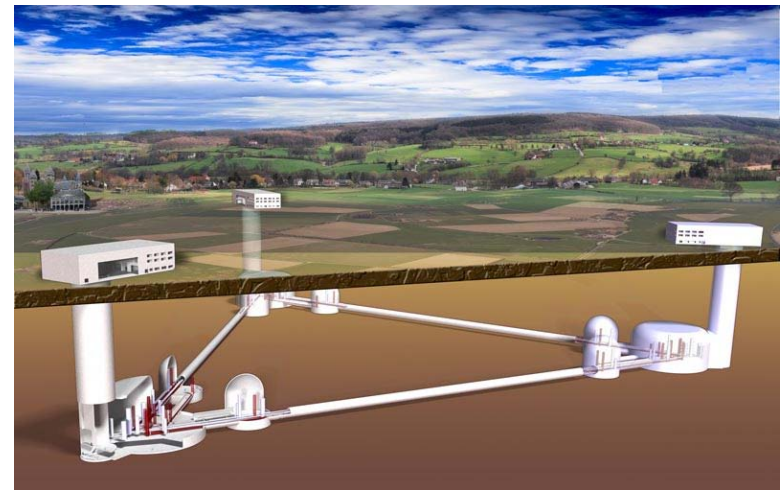
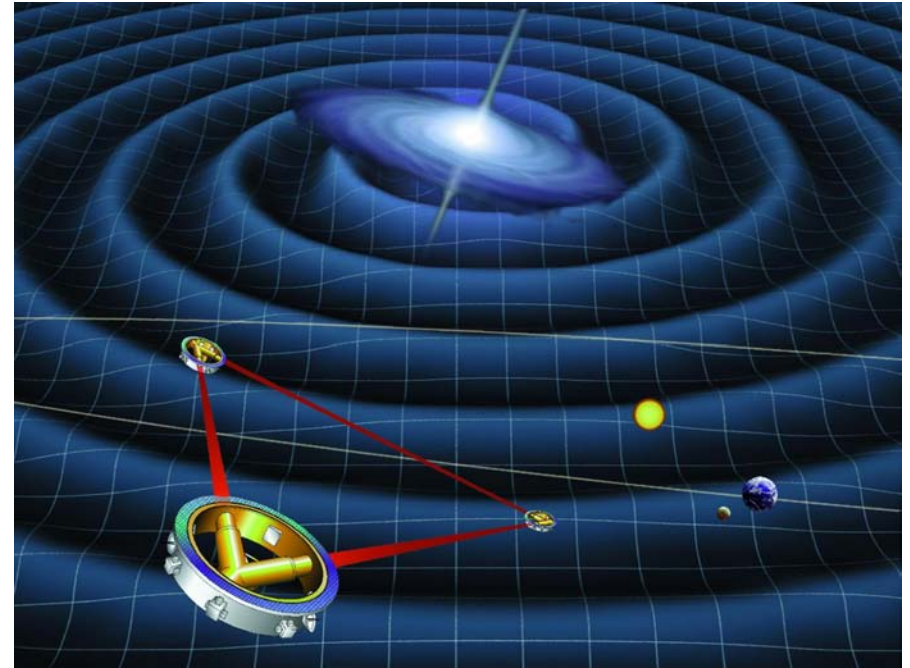
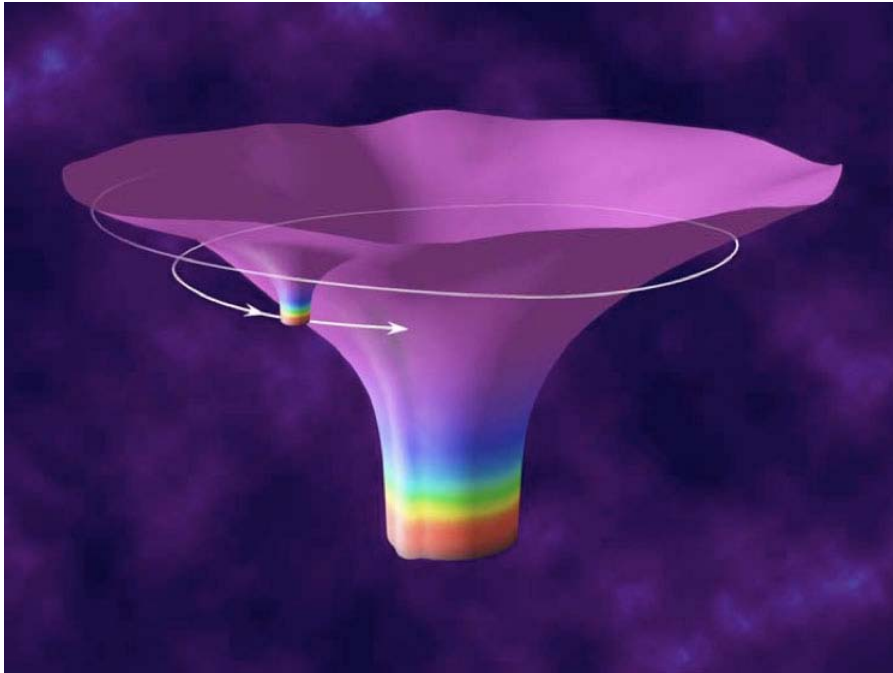
Energia Oscura



"Recording a gravitational wave [...] has never been a big motivation for LIGO, the motivation has always been to
open a new window to the Universe"

Kip Thorne (2016)

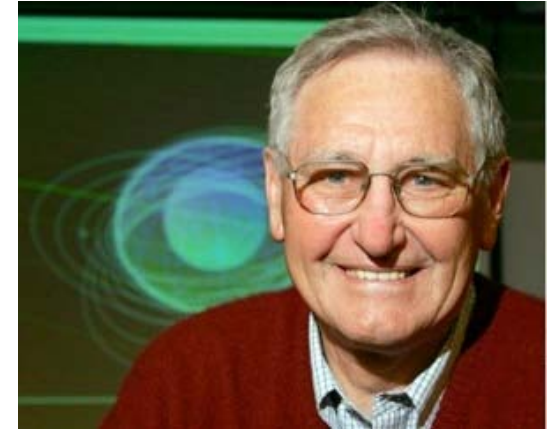
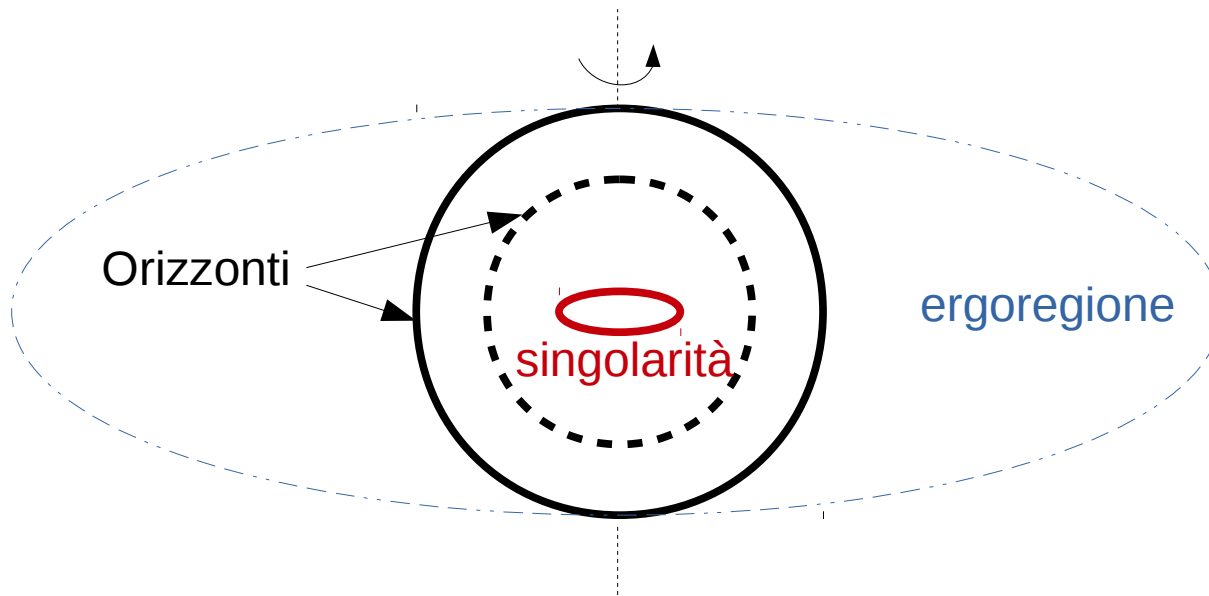
Prospettive future



Backup slides

*“Nothing is More Necessary
than the Unnecessary”*

Buchi neri di Kerr (1963)



- ◆ Curve chiuse di tipo tempo (paradosso del nonno)
- ◆ Unicità (Hawking, Penrose)

"In my entire scientific life [...] **the most shattering experience** has been the realization that an **exact solution of Einstein's equations** of general relativity, discovered by the New Zealand mathematician, **Roy Kerr**, provides the absolutely **exact representation of untold numbers of massive black holes that populate the universe.**

[S. Chandrasekhar]

Il trionfo di Einstein

LIGHTS ALL ASKEW IN THE HEAVENS

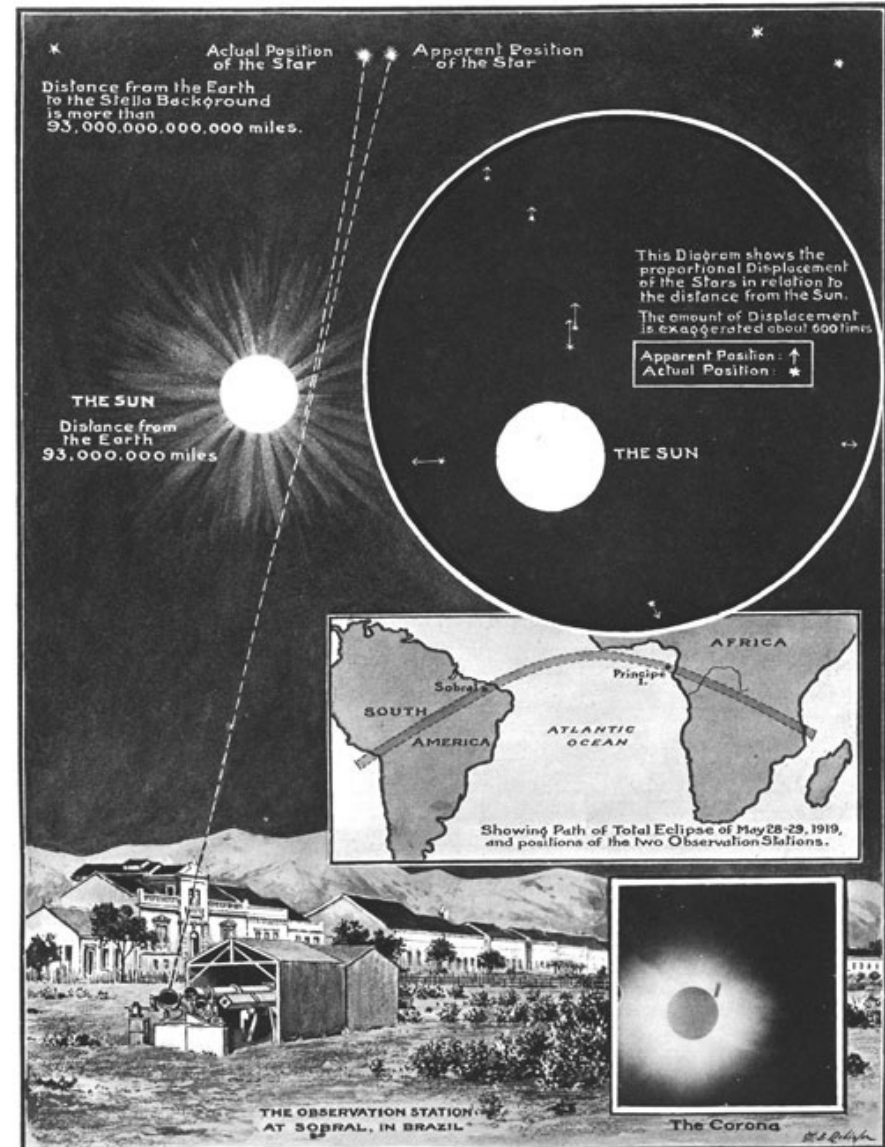
Men of Science More or Less
Agog Over Results of Eclipse
Observations.

EINSTEIN THEORY TRIUMPHS

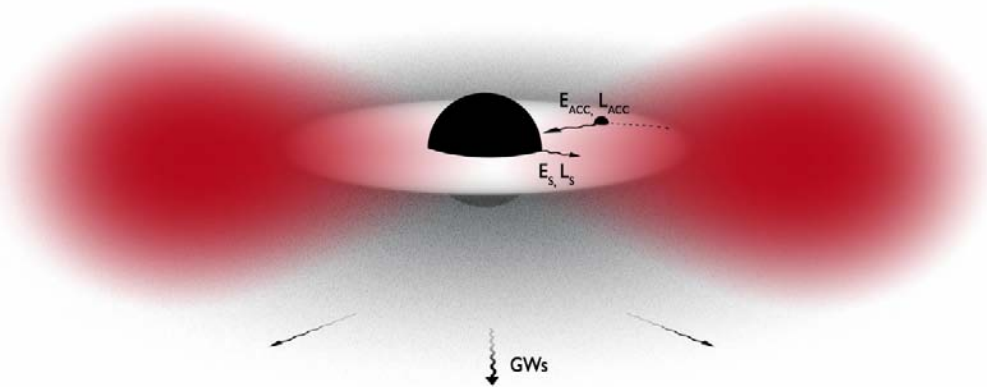
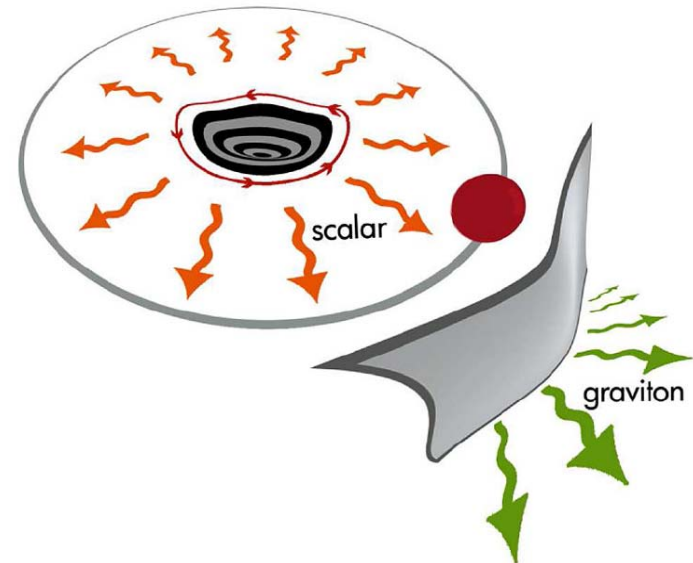
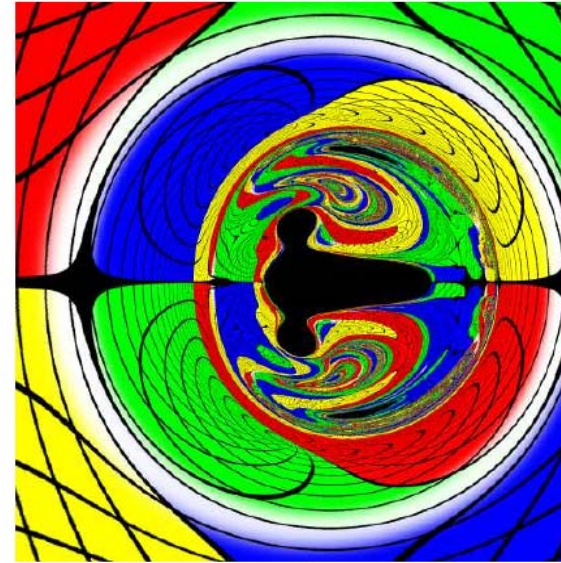
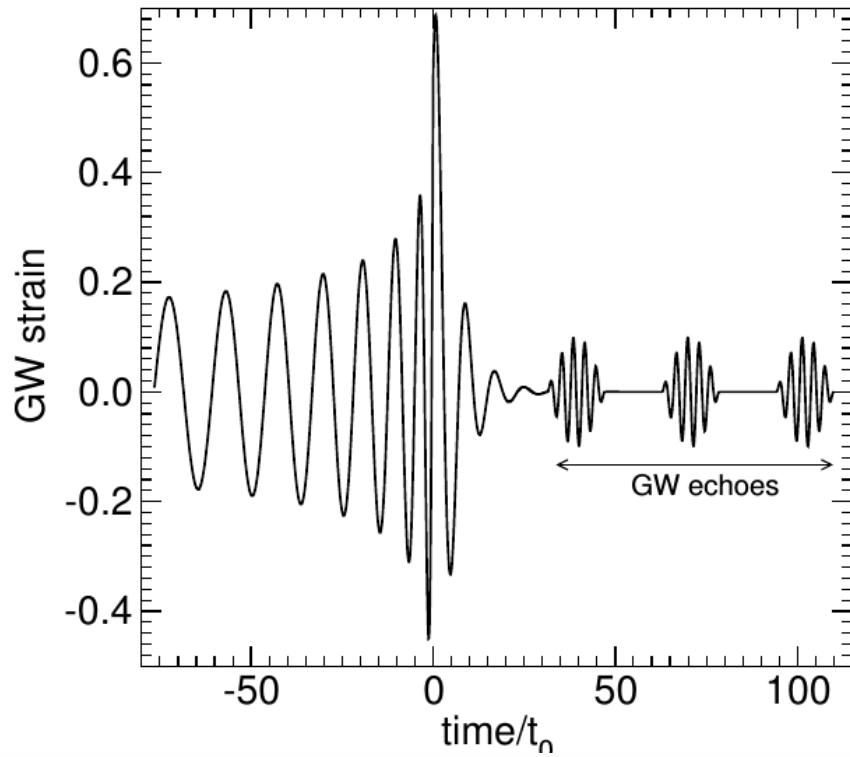
Stars Not Where They Seemed
or Were Calculated to be,
but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could
Comprehend It, Said Einstein When
His Daring Publishers Accepted It.



Hic Sunt Leones (2)



Binary Black Hole Evolution:
Catech/Cornell Computer Simulation

Top: 3D view of Black Holes
and Orbital Trajectory

Middle: Spacetime curvature:
Depth: Curvature of space
Colors: Rate of flow of time
Arrows: Velocity of flow of space

Bottom: Waveform
(red line shows current time)

